Math 129 (Fall '05) - Sample Exam 2 - Kennedy

WARNING: This is a past exam I gave. You should not assume that if a certain topic does not appear here it will not appear on our exam.

1. (14 points) Consider the area enclosed by the y-axis, the x-axis, the vertical line $x = 1$ and the curve $y = e^{-x}$. It is rotated about the y-axis. Find the volume of the resulting solid. For full credit you must do the integral analytically. However, a numerical answer is better than nothing.

2. (14 points) A water truck weighs 10,000 lbs when it is full of water. The truck starts up a mountain road full of water. The truck travels at a constant speed and the road has a constant incline. At the start of the trip the truck springs a leak. Water leaks out at a constant rate and at the top the truck only weighs 6,000 lbs. The top of the road is 5,000 feet higher than the bottom.
(a) Find a formula for the weight of the truck as a function of its elevation h assuming that $h = 0$ where the truck starts its trip.

(b) Find the total work done by the truck.

3. (8 points) Find the arc length of the graph of $y = 2x^{3/2}$ for $0 \leq x \leq 3$.

4. (14 points) A dam is 200 feet across the top and 100 feet tall at its midpoint. Its shape is approximately given by the parabola $y = x^2/100$ with $-100 \leq x \leq 100$. The water behind the dam goes up to the very top of the dam. Find the total force on the dam. (Recall that at a depth of h feet below the surface the water pressure is $62.4h$ lbs per ft^2

5. (14 points) A cylindrical barrel is 5 ft tall has a radius of 1.5 ft. It it filled to a depth of 4 ft with a mysterious liquid whose density depends on the depth in the liquid. The density d depends on the distance h below the surface according to $d(h) = 40(1 + h/10)$. (The density is in lbs/ft^3 and the distance h is in feet.) Find the total work needed to pump the liquid to the top rim of the barrel.

6. (12 points) Find the sums of the following series

$$1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} \cdots \frac{1}{2^{20}}$$

$$-3 + 1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} \cdots$$

$$\sum_{n=0}^{\infty} x^{2n}$$

7. (14 points) A tetrahedron has vertices at $(0,0,0)$, $(2,0,0)$, $(0,1,0)$ and $(0,0,1)$. Find its volume. (A tetrahedron has four faces, each of which is a triangle. Moreover, any slice through a tetrahedron is a triangle.)

8. (10 points) Determine whether the following improper integral converges and explain your reasoning.

$$\int_0^\infty e^{-x}\,(1+\cos x)\,dx$$

There are lots of ways to do this. Grading will be based on how well you explain your reasoning.