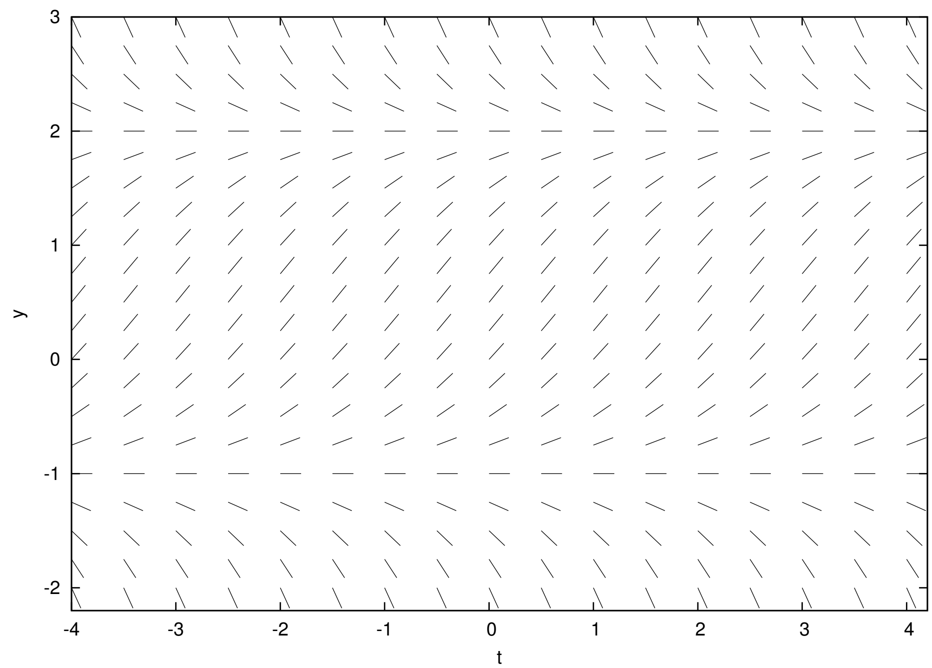
Math 129 (Fall '05) - Sample Exam 3 - Kennedy

WARNING: This is a past exam I gave. You should not assume that if a certain topic does not appear here it will not appear on our exam.

1 The slope field for $\frac{dy}{dt} = 0.5(1+y)(2-y)$ is shown below.

(a) Sketch the solution which goes through the origin. (Your sketch should include the solution for negative values of t.)

(b) Consider the initial condition $y(0) = c$. For what values of c will the solution $y(t)$ converge to 2 as t goes to ∞ ?



2. Find the radius of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{2^n x^n}{n} \tag{1}$$

3. Find the Taylor series of e^x about $x = 1$. (NOTE that it is about $x = 1$, not about $x = 0$.) Your answer should go up to at least the $(x-1)^3$ term.

4. For each of the following series state whether it converges or not and give the name of the test that gives your answer (integral test, comparison test, ratio test, alternating series test). In the case of the comparison test you should say what the series is that you are comparing with.

$$\sum_{n=1}^{\infty} \frac{n}{2^n} \tag{2}$$

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \tag{3}$$

$$\sum_{n=1}^{\infty} \frac{1+\cos(n)}{e^n} \tag{4}$$

5. Consider the differential equation and initial condition.

$$\frac{dy}{dt} = te^y, \quad y(0) = 0 \tag{5}$$

(a) Use Euler's method with $\Delta t = 0.1$ to find $y(1)$.

(b) Find the exact solution of the differential equation. As a check on your answer, compare your exact value for $y(1)$ with your answer in (a).

6. (a) Approximate $\sin(0.2)$ using a fifth-degree Taylor polynomial about $x = 0$ for $\sin(x)$.

(b) Estimate the magnitude of the error in your approximation.

(c) Now approximate $\sin(0.2)$ using a sixth-degree Taylor polynomial about $x = 0$ for $\sin(x)$ and estimate the magnitude of the error in your approximation.

7. (a) Find the Taylor series of $\tan^{-1}(x)$ about $x = 0$. Hint : what is the integral of $1/(1 + x^2)$?

(b) Use your answer to (a) to find the value of the tenth derivative of $\tan^{-1}(x)$ at $x = 0$.