Families of Functions

1. $f(x) = x^3 - ax$

A. For positive values of a, answer the following:

    i) How many critical points does $f(x)$ have? Find the coordinates of all local maximums
       and minimums. Your answer will be in terms of a.

    ii) How does increasing the value of a affect the position of the critical points? Illustrate by
       sketching 3 well-chosen family members, indicating the value of a for each.

B. For negative values of a, answer the following:

    i) How many critical points does $f(x)$ have? ? Find the coordinates of all local maximums
       and minimums. Your answer will be in terms of a.

    ii) How does decreasing the value of a affect the shape of the graph? Illustrate by
       sketching 3 well-chosen family members, indicating the value of a for each.

C. Find the value of a so that $f(x) = x^3 - ax$ has a local maximum value of 5.

2. $g(x) = x^6 - bx^4 + c$

A. For positive values of b, answer the following:

   i) How many inflection points does $g(x)$ have?

   ii) How does increasing the value of b affect the shape of the graph and the position of the inflection points? Illustrate by sketching 3 well-chosen family members, indicating the value of b for each.

B. For negative values of b, answer the following:

   i) How many inflection points does $g(x)$ have?

   ii) How does decreasing the value of b affect the shape of the graph and the position of the inflection points? Illustrate by sketching 3 well-chosen family members, indicating the value of b for each.

C. Find values of b and c so that (1,10) is an inflection point of $g(x) = x^6 - bx^4 + c$.