COMPLETING THE SQUARE

Here are some examples to review the method of completing the square.

$$\begin{aligned} x^2+bx+c &= x^2+bx+\frac{b^2}{4}-\frac{b^2}{4}+c \\ &= \left(x^2+bx+\frac{b^2}{4}\right)-\frac{b^2}{4}+c \\ &= \left(x+\frac{b}{2}\right)^2-\frac{b^2}{4}+c \end{aligned}$$

$$\begin{aligned} x^2-6x+7 &= x^2-6x+9-9+7 \\ &= \left(x^2-6x+9\right)-9+7 \\ &= \left(x-3\right)^2-2 \end{aligned}$$

$$\begin{aligned} ax^2+bx+c &= a\left(x^2+\frac{b}{a}x\right)+c \\ &= a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}\right)+c \\ &= a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)-a\cdot\frac{b^2}{4a^2}+c \\ &= a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}+c \end{aligned}$$

$$\begin{aligned} 2x^2+10x+3 &= 2\left(x^2+5x\right)+3 \\ &= 2\left(x^2+5x+\frac{25}{4}-\frac{25}{4}\right)+3 \\ &= 2\left(x^2+5x+\frac{25}{4}\right)-2\cdot\frac{25}{4}+3 \\ &= 2\left(x+\frac{5}{2}\right)^2-\frac{19}{2} \end{aligned}$$

LONG DIVISION

Here are some examples to review long division of polynomials.

$$\frac{x^4+4x^2+5}{x^2+1}=x^2+3+\frac{2}{x^2+1}$$

$$\begin{array}{r} x^2+3 \\ x^2+1\overline{)\,x^4+4x^2+5} \\ \underline{-\left(x^4+x^2\right)} \\ 3x^2+5 \\ \underline{-\left(3x^2+3\right)} \\ 2 \end{array}$$

$$\frac{8x^3+2x+1}{2x+1}=4x^2-2x+2-\frac{1}{2x+1}$$

$$\begin{array}{r} 4x^2-2x+2 \\ 2x+1\overline{)\,8x^3+2x+1} \\ \underline{-\left(8x^3+4x^2\right)} \\ -4x^2+2x+1 \\ \underline{-\left(-4x^2-2x\right)} \\ 4x+1 \\ \underline{-\left(4x+2\right)} \\ -1 \end{array}$$