IMPROPER INTEGRALS (no closed form)

1. Use graphs to answer the following questions.

If
$$\int_{a}^{\infty} f(x)dx = K$$
 and $0 < g(x) \le f(x)$, what can we say about $\int_{a}^{\infty} g(x)dx$?

If $\int_{a}^{\infty} f(x)dx = K$ and $0 < f(x) \le g(x)$, what can we say about $\int_{a}^{\infty} g(x)dx$?

If $\int_{a}^{\infty} f(x)dx$ diverges and $0 < f(x) \le g(x)$, what can we say about $\int_{a}^{\infty} g(x)dx$?

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2. Suppose we have an improper integral whose integrand does not have an antiderivative that can be expressed in closed form. Use the results from problem 1 to create a strategy for proving convergence or divergence of such an improper integral.

3. Determine if the following families of improper integrals converge or diverge.

A.
$$\int_{a}^{\infty} \frac{1}{x^{p}} dx$$

B.
$$\int_0^a \frac{1}{x^p} dx$$

4. Make a list of helpful inequalities or strategies for finding larger/ smaller functions.