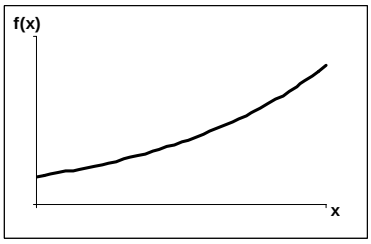
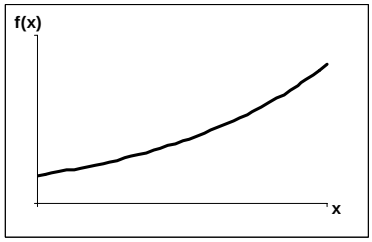
ESTIMATION TECHNIQUES

1. Assume we are trying to estimate the value of $\int_A^B f(x)dx$. Illustrate the indicated rule with $n = 2$ in each diagram. Include a formula each estimate and a general formula for the rule.

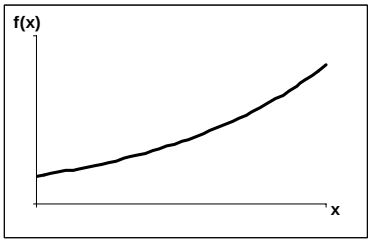
Left hand rule



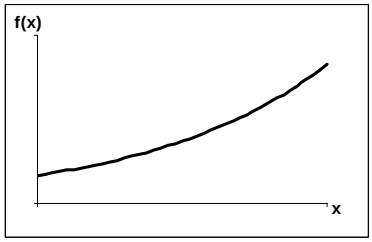
Right hand rule



Midpoint rule



Trapezoid rule



2. Complete the table using the words “overestimate” or “underestimate”.

	Shape of Graph			
Rule	Increasing Concave Up	Increasing Concave Down	Decreasing Concave Up	Decreasing Concave Down
Left hand				
Right hand				
Midpoint				
Trapezoid				

3. Suppose we estimate $\int_A^B f(x)dx$ using our rules with the same number of subdivisions, *n* but only record three of our estimates: $Right(n) = 1.8569$ $Mid(n) = 2.3481$ $Trap(n) = 2.1627$.
If $f(x)$ is monotone and does not have any inflection points in the interval [*A*, *B*],

A. Is $f(x)$ increasing or decreasing?

B. Is $f(x)$ concave up or down?

C. Estimate the value of $Left(n)$ and $Simp(n)$

4. The values in the tables below are for the estimates of $\int_0^2 e^{1.5x}dx$.

Estimates	Left	Right	Midpoint	Trapezoid	Simpson
N=15	11.49370621	14.03844447	12.70250984	12.76607534	12.7236983
N=75	12.47091390	12.97986156	12.72284308	12.72538773	12.7236913
N=375	12.67286438	12.77465391	12.72365735	12.72375914	12.7236913

Errors	Left	Right	Midpoint	Trapezoid	Simpson
N=15	-1.229985072	1.314753185	-0.02118144	0.042384056	7.0603 E-06
N=75	-0.252777379	0.256170273	-0.00084821	0.001696447	1.1309 E-08
N=375	-0.050826906	0.050962625	-3.3930 E-05	6.78596 E-05	1.8208 E-11

Ratio of Errors	Left	Right	Midpoint	Trapezoid	Simpson
N=15 to N=75	0.205512558	0.194842861	0.04004480	0.040025592	0.00160182
N=75 to N=375	0.201073790	0.198940430	0.04000179	0.040001020	0.00160996

A. What is the relationship between the errors in the Midpoint and Trapezoid rules?

B. Find a pattern for the error using each rule (express as a formula).

C. What characteristic of $f(x)$ determines the size of the errors in the Left and Right rules?

D. What characteristic of $f(x)$ determines the size of the errors in the Midpoint and Trapezoid rules?

5. Suppose $Mid(10) \approx 35.619$ and $Mid(20) \approx 35.415$. Find an estimate of the error when using $Mid(10)$. Use this information to find a better estimate for the value of the corresponding definite integral.