

1. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$ converges by comparing it to the improper integral $\int_1^{\infty} \frac{1}{x^4} dx$.

2. Show that the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by comparing it to the improper integral $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$.

3. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges for $0 < p < 1$.

4. Use the integral test to determine if the series converge.

A. $1 + \frac{1}{5} + \frac{1}{9} + \frac{1}{13} + \cdots + \frac{1}{4n-3} + \cdots$

B. $\frac{1}{2} + \frac{2}{5} + \frac{3}{10} + \frac{4}{17} + \cdots$

5. A. Does the series $\sum_{k=1}^{\infty} \frac{1}{k^5}$ converge? You must give a mathematically valid reason for your answer.

B. Does the series $\sum_{k=1}^{\infty} \frac{1 + \cos(k)}{k^5}$ converge? You must give a mathematically valid reason for your answer.