| Specific Example | General |
|---|---|
| **The problem:**<br><br>Find the area of the irregular shaped region bounded by $f(x)$, the $x$-axis, over the interval $[a, b]$. | **The problem:**<br><br>The quantity we want to find depends on something that varies. Applications come from geometry (length, area, volume), biology (population), physics (work), or anything that can be measured. |
| **Creating a slice:**<br><br>Look at the area of the region bounded by $f(x_i)$, the $x$-axis, over the interval $[x_i, x_i + \Delta x]$. A rectangle can be used to estimate this irregular region. The rectangle is called a slice.<br><br>area of the rectangle $= f(x_i)\Delta x$ | **Creating a slice:**<br><br>Take a slice of the problem. We assume that everything is constant over that slice so that we can use formulas from algebra or trig. In geometry the slice might be a rectangle, a cylinder, or a box. In applied fields the slice might only depend on the units involved. Once the slice is determined, a formula is developed to approximate the quantity of interest. Don't forget, $\Delta x$ has units too. |
| **Setting up a Riemann Sum:**<br><br>Divide the interval $[a,b]$ into $n$ equal subintervals. Create $n$ rectangles and add up their areas.<br><br>sum of areas of rectangles $= \displaystyle\sum_{i=1}^{n} f(x_i)\Delta x$<br><br>If only an estimate of the area is needed, we can stop at this step. | **Setting up a Riemann Sum:**<br><br>Divide the interval of interest into $n$ equal sub-divisions. The Riemann Sum will depend on the formula we developed for a single slice. If we only want an estimate of the quantity of interest, we can stop at this step.<br><br>A left hand rule will start with $i = 0$ and end with $i = n-1$. A right hand rule will start with $i = 1$ and end with $i = n$. |
| **Creating a definite integral:**<br><br>Take the limit of the Riemann Sum as $\Delta x \to 0$.<br>$\displaystyle\lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)dx$<br><br>Use the Fundamental Theorem to evaluate the definite integral.<br><br>$\displaystyle\int_a^b f(x)dx = F(a) - F(b)$ | **Creating a definite integral:**<br><br>We take the limit of the Riemann Sum as $\Delta x \to 0$. The expression involving the limit is called the definition of the definite integral. If we want to evaluate it, we use the Fundamental Theorem and integration techniques. In some cases, we cannot find the closed form for $F(x)$. In that case we can only find a Riemann Sum. |