Name $\qquad$

USE WELL-KNOWN SERIES TO ANSWER THE FOLLOWING.

1. Find $3+\frac{27}{3!}+\frac{243}{5!}+\frac{2187}{7!}+\ldots$.
2. Find $x^{2}-\frac{x^{4}}{3!}+\frac{x^{6}}{5!}-\frac{x^{8}}{7!}+\ldots$.
3. Find $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^{k}}{k}$.
4. Use series to find $f^{(5)}(0)$ and $f^{(6)}(0)$ for $f(x)=\frac{x}{1-x^{2}}$.
5. Use the values in the table below to find the limits. Show work to justify your answer. In other words, what does this have to do with Taylor polynomials?
A. $\lim _{x \rightarrow 2} \frac{f(x)}{h(x)}$ and
B. $\lim _{x \rightarrow 2} \frac{f(x)}{g(x)}$.

|  | Function Value at <br> $x=2$ | First Derivative <br> Value at $x=2$ | Second Derivative <br> Value at $x=2$ |
| :--- | :---: | :---: | :---: |
| $f(x)$ | 0 | 0 | 3 |
| $g(x)$ | 0 | 22 | 5 |
| $h(x)$ | 0 | 0 | 7 |

6. Use the series for $\ln (1-x)$ and differentiation to find a series for $\frac{1}{1-x}$.
7. Use the series for $\frac{1}{x^{2}+1}$ and integration to find a series for $\arctan x$.
8. Find a series for $\int_{0}^{x} t e^{t} d t$.
9. In this problem you will evaluate/ approximate $\int_{0}^{1} \sqrt{2-x^{2}} d x$ in four different ways.
A. Use the first two nonzero terms of an appropriate series to get an approximation.
B. Use Simpson's rule with $n=20$ to get an approximation.
C. Break up the region into a triangle and a part of a circle, then use geometry to get an exact value.
D. Use the integration tables to get an exact value.
