Universality and conformal invariance in random walks

This RTG is about random walks and random self-avoiding walks (SAW's) in two dimensions. In both cases the walk takes place on a lattice like $\mathbb{Z}^2$. In the ordinary random walk we choose one of the four lattice directions at random and take a step in that direction. We repeat this process. The precise definition of the self-avoiding walk is more involved, but the result is a random walk that never visits any lattice site more than once. We are mainly interested in the scaling limits of these models. This means that we use a lattice with lattice spacing δ and try to understand what happens when $\delta \to 0$.

The ordinary random walk is very well understood. In particular we know what the scaling limit is - Brownian motion. We also know that the scaling limit has a certain invariance under conformal maps. By contrast there are virtually no theorems about the self-avoiding walk. In the last decade there has been an explosion of conjectures about the self-avoiding walk. In particular, we now have a candidate for the scaling limit - a stochastic process called Schramm-Loewner Evolution (SLE). We will give an introduction to SLE and how it is believed to describe the SAW. The SAW also behaves nicely under conformal maps, so this RTG will involve some complex analysis in addition to probability.

One can do Monte Carlo simulations of the SAW. Until recently we only knew how to do these simulation in a few geometries. For these special cases they show excellent agreement with the SLE predictions. In the past year we have discovered how to do these simulations for a much broader class of geometries. A possible project for the Fall is to test the the SLE predictions for this broader class of geometries.