

# Eigenvalues and their multiplicities

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*Abstract.* Eigenvalues of a symmetric matrix with real entries are real, and generically all eigenvalues are different. What is the codimension of the set of symmetric matrices with multiple eigenvalues in the space of all symmetric matrices? The answer looks almost obvious: a matrix has multiple eigenvalues if the discriminant of its characteristic polynomial vanishes; this discriminant is a function of the entries of the matrix, so all matrices with multiple eigenvalues are solutions of one equation, and they should form a set of codimension 1 in the space of all matrices. However, this obvious answer is wrong. The correct answer is 2. One can compute the codimension of the set of matrices with a given degeneration of eigenvalues, say three double eigenvalues, one triple eigenvalue, etc.. In his 1967 paper "Modes and Quasimodes", Arnold made bold conjectures that the same answers hold in infinite-dimensional situation, for differential operators. Some of his conjectures are theorems now, but most of them are still wide open. I will discuss Arnold's conjectures. I will also discuss some bounds for eigenvalues of differential operators that arise in geometry.