Solve only one of the following two problems.

1A. Compute the following integral:
\[
\int_0^\infty \frac{\cos(x)}{1+x^4} \, dx.
\]

1B. Find a conformal mapping of the vertical semi-infinite strip \( \{0 < \text{Re}(z) < 1, \text{Im}(z) > 0\} \) onto the unit disc \(|w| < 1\).

2. Compute the singular homology groups \( H_*(X, \mathbb{Z}) \) of the space \( X = \mathbb{R}^3 \setminus A \), where \( A \) is a subset of \( \mathbb{R}^3 \) homeomorphic to the disjoint union of two unlinked circles.

3. Consider the following map \( f : \mathbb{R}^3 \to \mathbb{R}^2 \):
\[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\mapsto
\begin{pmatrix}
\frac{xz - y^2}{y - z^2} \\
yz - x^2
\end{pmatrix}.
\]
For which values \((a, b) \in \mathbb{R}^2\) of \( f \) is the level set \( f^{-1}(a, b) \) a smooth submanifold of \( \mathbb{R}^3 \) ?

4. Consider the surface \( \Sigma \) obtained by identifying the edges of a square in the following way:

(a) Construct a model of the universal covering space of this surface, indicating especially how \( \pi_1(\Sigma, v) \) acts.
(b) Identify the covering space \( X \) of \( \Sigma \), which corresponds to the subgroup of \( \pi_1(\Sigma, v) \) generated by \( a \) and describe the group of covering automorphisms of \( X \).

5. Consider the submanifold \( \iota : M \hookrightarrow \mathbb{R}^3 \) given by \( x^2 + y^2 - z^2 = 1 \).
(a) Show that the vector field \( X = \frac{xz}{1 + z^2} \frac{\partial}{\partial x} + \frac{yz}{1 + z^2} \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \) is tangent to \( M \), i.e. that there exists a vector field \( Y \) on \( M \) such that for any \( m \in M \) we have \( \iota_*(Y(m)) = X(m) \).
(b) Show that the two-form \( \omega = x \, dy \wedge dz + y \, dx \wedge dz + z \, dx \wedge dy \) restricts to an area form on \( M \), i.e. a two-form which never vanishes. (Hint: use cylindrical coordinates.)
(c) Does the flow of \( Y \) on \( M \) preserve \( \iota^*(\omega) \)?

6. Prove the Poincaré lemma in the plane: a closed 1-form or 2-form on \( \mathbb{R}^2 \) is exact.

7. Let \( T^2 \) be the two-dimensional torus and let \( \phi : S^2 \to T^2 \) be a smooth map. Show that for any top de Rham cohomology class \([\nu] \in H^2(T^2)\), we have \( \phi^*[\nu] = 0 \).