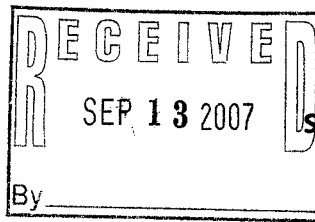


VIGRE Funding Report

(due 30 days after semester of support)



Semester/Summer and Year:

Summer 07

Name: Cameron McLeman

List the graduate courses you have taken this semester (including independent studies), your grades, and the instructors:

Course	Title	Grade	Instructor

List the title, date and location of any talks you have given, either here or elsewhere:

If you are working on your dissertation, include a one paragraph description of your research progress. If you have not yet begun dissertation research, describe your progress toward finding a dissertation topic and advisor and beginning that research.

I significantly furthered my research this summer by dramatically increasing my bound on the size of the Galois group of a p -class field tower of a certain type. Roughly speaking, at the beginning of the summer I was able to show this Galois p -group (p odd), if indeed one of its kind exists, would have to have order at least p^7 . This summer, I was able to raise this bound dramatically to show that not only must it have order at least p^{20} , but its commutator subgroup must have order at least p^{18} . This is at least heuristic evidence that such a group is unlikely to occur.

List publications, if any.

Check all activities you completed during the funded period:

Academics:

- Independent Study
- Oral Comprehensive Exam
- Commence Thesis Research
- Conference attendance
- Conference participation
- Complete PhD

Professional development and outreach:

- AP Calculus Visit
- High School Workshops
- Undergraduate Research Project
- Undergraduate Research Seminar
- Super TA
- Mentoring junior graduate students for the qualifying exams
- RTG (help organize)
- Research Seminar (help organize)

Other (please specify)

I was a senior graduate student at the Summer 2007 Integration Workshop for incoming graduate students.

Attach a brief statment about your academic progress and professional development during the period of support.

VIGRE REPORT

CAM MCLEMAN

I spent the majority of the summer working on improving the results of my research. Without going into the details (more of which were included in my VIGRE proposal), I was interested in studying the Galois group G of the Hilbert p -class field tower over a quadratic imaginary number field. In the case I am studying (indeed, the only case where the finiteness of $|G|$ is currently still open), it is known that G can be given a presentation $G = \langle x, y | w_1 = w_2 = e \rangle$ as a pro- p -group. The two defining words w_1 and w_2 have been shown to lie in very particular levels of the so-called Zassenhaus filtration $F_{(n)}$ of the free pro- p -group on two generators. Namely, $w_1 \in F_{(3)}$ and w_2 must lie in one of $F_{(3)}$, $F_{(5)}$, or $F_{(7)}$. All of this is proven using the famous Golod-Shafarevich theorem, which also implies that if there is a finite such group with $w_2 \in F_{(7)}$, it must have some remarkable properties. My research has been to enumerate some of these properties, and as a conclusion, arrive at a lower bound for the size of this group. This summer, this bound has increased significantly (compare to the result in my proposal). The strongest version of my theorem will require the notation $d_n = \dim_{\mathbb{F}_p} G_{(n)}/G_{(n+1)}$, and now reads:

Theorem. *Suppose $p > 7$ and let G be the Galois group of the p -class field tower over a quadratic imaginary number field K . Suppose that the p -class group of K has rank 2 (the only unsolved case), so is isomorphic to $\mathbb{Z}/p^a\mathbb{Z} \oplus \mathbb{Z}/p^b\mathbb{Z}$ for some $1 \leq a \leq b$. Then $d_1 = 2$, and $d_2 = d_3 = d_4 = 1$. Further, we have $|G| \geq p^{17+2a+b}$. In particular, we have $|G| \geq p^{20}$ and $||G, G|| \geq p^{17+a}$.*

On a more meta-mathematical level, I have also spent much time writing my thesis. A rough draft is now complete, and I am working on editing, and on writing the various forms of summaries (for the NSF, for job applications, for letter-writers, etc.) that I will need in the upcoming year. I have a rough draft of a teaching statement, and have begun to organize myself for the job application process. Finally, on the vertical integration front, I participated as a senior graduate student for the Summer 2007 Integration Workshop for incoming first-year graduate students in mathematics.