The N-body Problem

The realms of Newtonian mechanics:

A. Space in which motion takes place is three dimensional Euclidean space E^3 with fixed orientation. Time is one dimensional R.

B. Newton-Laplace Principle of Determinacy: The state of mechanical system at any fixed moment of time uniquely determines all of its (future and past) motion.

Example: N-body problem. One considers n particles (q_i, m_i) in E^3. By the law of universal gravitational (attraction):

$$m_i \ddot{q}_i = \frac{\partial U}{\partial q_i} \qquad (1)$$

Collisions and Regularizations

If two or more particles come together at a certain time then the solution is said to experience collision. At such time the potential energy is approaching to infinity, the equations of motion become undefined and the solution has a singularity.

Example of regularization: Levi-Civita Regularization of the Kepler problem.

Analytic Versus Block Regularization

Definition 1 *A solution is well behaved if R is its interval of existence. Otherwise we way the solution is singular.*

Classical question: Can we connect two singular solutions?

This leads to the notion of analytical regularization. Note that we tacitly assume that a solution's dependence on time is of primary importance while its functional dependence on initial conditions is secondary. Yet there is another kind of regularization which arises when dependence initial conditions is given primary importance. We will refer to this kind of regularization as a block regularization. It is firstly introduced by Robert Eation 1970.

McGehee Transform

Question: Are triple collisions in the collinear three body problem block regularizable?

In order to answer this question McGehee has introduced a transformation (change of coordinates) replacing singularity set with the manifold.

By studying geometrically and topologically the flow on this manifold he was able to answer the question.

Answer: There exists values of masses for which three body problem in not block regularizable.

Simultaneous Binary Collisions

Question: What happens if the several collisions occurs simulataneosly in the N-body problem?

The simplest case of this phenomenon occurs in the collinear 4-body problem when two binary collisions occurs simultaneously. First partial results of this type due to Belbruno. More general result:

Theorem 2 (C. Simo) *Simultaneous binary collisions in the classical n-body problem are time continuation regularizable(therefore block regularizable).*

Question 1: Are simultaneous binary collisions analytically regularizable?

Question 2: What is the regularizing transformation?

Sigel's Results

Sundman showed (almost a century ago) that double collisions can be analytically regularized in the two body problem. Since double collisions can be regularized, one is led to ask whether the same can be done to other singularities. Siegel has addressed this question for triple collision in the three-body problem. He found that the most solutions cannot be extended through triple collision as analytic functions of some transformed time variable. He also showed that the set of orbits in triple collision forms a smooth submanifold of the phase space.

Do We Have the Answers?

We don't know how to find regularizing transformation. May be the one doesn't exist. We used McGehee transform actually its modified version to write equations of motion. Following Sigel's program, we linearized the flow and hoped that the eigenvalues were not rational numbers therefore showing that the singular points are essential singularities and cannot be regularized. However we found no obstruction to analytic regularization of binary collisions and we are lead to believe the following:

Conjecture 3 *There exists regularizing transformation such that simultaneous binary collisions in collinear 4-body problem are analytic regularizable.*

At this point we are unable to come up with this transformation.