VIGRE REPORT- Fall 2003

Sarah E. Frey

Dissertation Progress

This semester I have primarily focused on continuing laying the foundations for my understanding and research in non-linear elasticity theory. I have been working on problems given to me by Dr. Tabor to develop my skills in this field. Throughout the semester, I met weekly with Dr. Tabor, Dr. Goriely and Dr. Tongen to discuss the applications of elasticity theory that each of us was pursuing. Specifically, I have been working on the problem of creating a non-linear elastic system to model the deformation of a sphere due to self-gravity. It is hoped that by studying this system and the stability of its solutions that some light may be shed on why instabilities are observed in the simplified linear elastic system for the tidal deformation of the sphere. At this point, I have a system set up but am still searching for a way in which to solve this complicated non-linear system of equations.

I have also been working on polishing the results I obtained over the summer for the tidal case in which density is radially dependent. I have run some more cases of my code and generated new plots that illustrate the singular behavior of the solutions. I have also began writing up these results for publication, which will hopefully be ready for submission some time in February.

Finally, I have just started working on the creation of a time dependent numerical model for the tidal deformation of a sphere. It is hoped that this time dependent simulation will allow us to gain greater insight as to the runaway behavior and cause of this instability. I have been looking into the program FEMLAB in hopes of exploiting some of its predesigned grids and finite element schemes in this modeling.

This semester I also had two papers which I had written previously,accepted for publication. The citations for these papers are given below.

"Numerical Evaluation of Love's Solution for Tidal Amplitude: Amplified tidal effects near singularities" with T.A. Hurford and R. Greenberg.
Published in the Proceedings of the IAU Conference on Tides, 2002.
Accepted for publication in Celestial Mechanics and Dynamical Astronomy, 2003.

"Data Assimilation with Extended Kalman Filtering for Impact Shock-Wave Dynamics" with J. Kao, et. al..
Accepted for publication in the Journal of Computational Physics.

Courses Taken

Audited MATH 565A - Stochastic Processes taught by Dr. Watkins

Research Talks/ Seminars

"Instabilities in the Problem of Elastic Planetary Tides"; Applied Mathematics Brown Bag Seminar; Univ. of Arizona; September 5th, 2003

"Characterization of Instabilities in the Tidal Deformation of a Planetary Body"; Joint Mathematics Meeting, AMS Special Session on Celestial Mechanics; Pheonix, AZ; January 10th, 2004

Vertical Integration Activities

This semester I was in charge of running the high school workshop program. We did two workshops for high school classes and a special workshop for a group of talented middle school students. Attached is a copy of the full report on these activities.

This semester, I continued to help a group of 1st and 2nd year students with course work problems, specifically from the Methods and Perturbation Theory courses.

I have also begun serving as the records officer for the University of Arizona SIAM student chapter. This duty has involved helping with the planning of chapter events and keeping documentation of the chapter activities and meetings.

High School Workshop Summary

Knot Theory Workshop

Thursday, November 6th, 2003

This workshop was done for Linda Pearman's intermediate algebra class from Pueblo High School. There were 12 students, mostly sophomores, plus two instructors at the workshop. The graduate students involved in the workshop planning were Adam Spiegler, Sarah Frey, Brad Weir, Josh Chessler, Rob Pawlowski, Patrick Shipman, and Ben Levitt. The graduate students involved in running the workshop were Adam Spiegler, Sarah Frey, Brad Weir, Josh Chessler, Ben Levitt, Jeannine Smallwood, and Dmitry Kondrashov.

The students arrived at 8:15 and the workshop began with a brief overview by Sarah Frey. Brad Weir then gave an introduction to the theory of knots with a brief history of how the interest in the study of knots developed. Sarah Frey then lead an activity session on Crossing Numbers in which she introduced the idea of differentiating various knots and using crossing numbers to classify these different knots. The students did a worksheet on distinguishing unknots, trefoils and figure-eights using crossing numbers and a worksheet on identifying pentoils. Following this Josh Chessler introduced the students to Reidimiester Moves. He showed the students the three basic moves and then helped them do a worksheet in which they were asked to find a series of moves to transform a given knot into another given configuration.

After a short break, Dr. Bruce Bayley came into to talk to the students about math in general. The underlying goal was to provide some motivation to why studying math was interesting and to encourage students to consider pursing math related fields in college. He did a demonstration of the creation of a parabola by spinning a fluid and related this to telescope mirror creation.

Adam Spiegler then introduced the rules of three-colorability and led the students through the coloring a few knots. He also gave them a couple of worksheets in which they had to color a knot if it was possible or state that it was not three colorable. After a lunch break, Ben Levitt explained why it is often useful to make knots from sticks and then led the students through the creation of knots from sticks with plastic connectors. One goal for the students was to create a knot that was not the unknot with a minimum number of sticks. Finally, Dmitry Kondrashov ended the day by giving a talk on proteins and their relation to knot theory. In the talk, he provided a brief introduction to protein function and structure and then discussed more details about the structure and connection to knots.

The students left to return to school at 2:00. Before leaving they filled out a feedback survey on the days events. Overall, the students seemed to enjoy the day and found most of the material understandable. Most of the students enjoyed the 3-colorability and Bruce Bayley talk the best. They seemed to have the most trouble with the Reidimiester moves and most students said this was confusing and not very interesting. The teacher seemed pleased with the workshop, particularly about Dr. Bayley's talk and the way in which it provided a nice segway into her next unit on parabolas. She also remarked that she is going to try to arrange a trip to the campus mirror lab as a result of the workshop. Overall, the workshop seemed quite successful.

MATERIALS: plastic tubes with connectors, sticks, plastic connectors, crayons, rope

High School Workshop Summary

Biomath Workshop

Friday, November 14th, 2003

This workshop was done for Diann Porter's AP Calculus classes from Salpointe High School. There were 28 students, mostly seniors, plus the instructor at the workshop. The graduate students involved in the workshop planning were Dmitry Kondrashov, Sarah Frey, Rachel Labes, Jared Barber, Bharath Narayanan, and Ben Levitt. The graduate students involved in running the workshop were Dmitry Kondrashov, Sarah Frey, Rachel Labes, Jared Barber, Ben Levitt, Cameron McLeman, and Chis Rassmussen.

The students arrived at 8:20 and the workshop began with a brief overview by Sarah Frey. Dmitry Kondrashov then gave an introduction to proteins with a brief motivation of how mathematics is useful in studying proteins and other biology topics. Rachel then introduced the students to the field of combinatorics and had them do a couple of counting problems including the number of pennies needed to make a pyramid n rows high, the number of handshakes interchanged ina room of n people, and the number of outfits possible with 4 hats, 4 shirts and 4 pants. Dmitry then related this to proteins by discussing the number of possible proteins. For a simple example, supose a protein is 100 units long. Each unit is composed of an amino acid and there are 20 different amino acids. Find the number of possible proteins of this type.

The students then moved on to topology and knot theory. Ben presented a very basic definition of topology and then discussed some fundamentals of knot theory. He explained the idea of classifying knots and had the students compute some crossing numbers. He ended by explaining that it is often useful to make knots out of sticks and had them consider what the minimum number of sticks needed to create a nontrivial knot was. Once again, this session ended with Dmitry discussing the relationship between this material and proteins. He discussed protein structure and folding.

Next, the students switched to looking at harmonic oscillators to better understand the mechanics of protein motion. Sarah presented the students with the challenge of creating a clock to measure one minute out of a piece of string, a paperclip, and some tape. A contest was held to see who could create the most accurate clock. After this activity, the students experimentallyinvestigated the relationship between frequency and length, amplitude, and weight of the pendulum. The students then were given a lunch break.

In their regular math class, many of the students had been working on a project on solving the Rubik's Cube. The instructor asked me before the workshop if it would be possible to have a graduate student come and present the students with a brief introduction to how to solve the cube and the matheamtics principles behind it. Chris came in and gave a brief introduction to group theory and the idea of transformations using permutation notation. He showed the students how to label the corners of a cube and how to denote the effect of each Rubik's Cube move in permutation notation. He had the students come up with notation for each move and then gave them a list of some possible move combinations and had them find the net effect using permutation multiplication. The net goal was to find a move which interchanged two corners and left other corner fixed. The students were left with some moves to be looked at later in their free time.

The final session was on indirect measurement and statistics. Jared provided a short introduction to statistics and motivated the usefulness of statistics in science. He then had the students do an experiment in which they measured the area of a penny through indirect means.

They traced a penny several times on a 10cm*10cm piece of paper and then dropped a pencil on the paper 100 times. They recorded the number of times the pencil mark was inside one of the traced circles. The fraction of the total area can thus be approximated by the number of hits inside the circles, and an estimate of pi can be obtained from these statistics. Dmitry concluded this activity by discussing the way in which proteins are actually measured and studied and how in science it is often impossible to measure things directly and how we must often rely on statistics to gain information about our world.

Finally, Dmitry ended the day by giving a summary of the different areas of mathematics that were explored and their relationships to proteins. He presented some additional information about how proteins are modeled mathematically and the limitations of such modeling.

The students left to return to school at 2:20. Before leaving they filled out a feedback survey on the days events. Overall, the students seemed to enjoy the day and found a majority of the material understandable. Most of the students enjoyed the Rubik's Cube talk the best. This is not surprising since they already had prior knowledge on this subject and this was a topic that they had already expressed interest in learning more about. There were very mixed review on many of the other sessions. Some students commented very negatively on particular sessions, while others seemed to enjoy that particular subject. I think the nice thing about having so many different areas of math in the workshop and different types of activities was that a student did not have to spend too long on an area they did not enjoy and they got exposed to a large number of topics so had a chance to find an area that interested them. The teacher seemed somewhat pleased with the workshop overall. She particularly expressed gratitude for the Rubik's Cube speaker which will help her in designing future activities for her students. Overall, the workshop seemed quite successful.

MATERIALS: plastic tubes with connectors, sticks, plastic connectors, pennies, pens, rulers, 10cm*10 cm paper squares, string, paper clips, tape

High School Workshop Summary

Elasticity and Bridges Workshop

Monday, December 1st, 2003

This workshop was done for Bonnie Tamkin's middle school accelerated math class from Casas Christian School. There were 6 students, mostly 6th graders, plus the instructor at the workshop. Sarah Frey planned and lead the activities for the workshop.

The class arrived at 9:00 and departed at 11:00. Also included in their field trip were visits to other departments and a tour of the campus. The students were given a brief introduction to elasticity theory and its importance in real world problems. The students then performed a lab to investigate bridge buckling. The students cut sheets of paper and folded up an inch on each side along the longest side of the paper. The paper bridge was then suspended between two books with an overlap of an inch and a half on each side. A styrofoam cup was placed on the bridge and loaded with pennies until the bridge buckled and collapsed. The results were recorded. This process was repeated for bridges made out of 2, 3 and 4 sheets of paper. Students graphed their results and were asked to find a line of best fit for their data. A discussion then ensued about how to find this best fit line and how to make predictions using graphed data. Finally, the students were given an open ended problem to create a bridge out of a single sheet of paper that will hold as much weight as possible. A contest was held to determine which design was most resistant to buckling.

To end the short workshop, a panel of graduate students poke to the students and answered questions prepared ahead of time by the students. The panel consisted of Sarah Frey, Katrina Piatek-Jimenez, Ben Levitt, and Cameron McLeman. The panel members each gave a brief introduction about where they are from and what they do mathematically. The panel then fielded questions from the students, which the teacher had them write before the workshop.

Overall, the workshop seemed quite successful and the teacher seemed pleased with the content. For sixth graders, the students were surprisingly inquisitive and had some very insightful thoughts about the problems posed.

MATERIALS: paper, scissors, rulers, books, styrofoam cups, pennies, graph paper