

ADAM SPIEGLER
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VIGRE Report - Spring 2004

During the spring semester of 2004, I received a VIGRE grant. My plan of study was to continue my research with Dr. Flascka with the n -dimensional rigid body, and in particular, prepare for comprehensive oral exams. I studied mainly a paper by Tudor Ratiu titled *The Motion of the Free n -dimensional Rigid Body* which concentrates on recovering the equations of motion for a general n -dimensional free rigid body abstractly, with essentially no regard to the physical nature of the problem (if there is a physical nature to the problem in n -dimensional space. In a sentence, the paper exploits the symplectic structure of the cotangent bundle of the Lie group $SO(n)$ in order to find the equations of motion. I continued on with the problem, using the energy-Casimir method to perform stability analysis on the equilibrium solutions for the three dimensional rigid body. Next I attempted to do a similar analysis for the four dimensional case. I presented the topics above to my committee and passed my exam on May 10th of 2004.

In addition to my preparation for my oral exam, I continued on with my research with Dr. Flaschka. We noticed a connection between the various types of equilibrium and their associated stability and to the orbits of the weights in the root space of the Lie algebra $\mathfrak{so}(n)$. We noticed that when considering various degenerate cases imposed on the inertial map, the resulting classes of equilibria demonstrate similar behavior as orbits of the weights. Currently, I am exploring this connection, and hopefully it will become more apparent and ultimately I hope to categorize the different types of equilibria for the general n dimensional rigid body.

In order to satisfy the vertical integration component of the grant, I organized two workshops on knot theory for local high school students. I prepared some topics on knot theory which I have presented in the past for this workshop, and with the help of about five graduate students who also prepared some materials, I organized an approximately five hour workshop which we presented on two occasions in the spring of 2004. Overall, the feedback was extremely positive from the teachers, and I expect that they will continue to bring students to these workshops in the future. Some of the topics we presented included crossing number, Reidemeister moves, three-colorability test, and applications to science.