DEPARTMENT OF MATHEMATICS

VIGRE Funding Report

(due 30 days after semester of support)

Semester/Summer and Year: Fall 2008

Name: Joseph McMahon

List the graduate courses you have taken this semester (including independent studies), your grades, and the instructors:

Course	Title	Grade	Instructor
MATH 920	Dissertation	S	Goriely

List the title, date and location of any talks you have given, either here or elsewhere:

8 Sept. 2008: Remote connection, SSH, and VNC
Software Interest Group (SWIG) Seminar, Dept. of Mathematics, The University of Arizona

If you are working on your dissertation, include a one paragraph description of your research progress. If you have not yet begun dissertation research, describe your progress toward finding a dissertation topic and advisor and beginning that research.

During the fall 2008 semester, I continued to develop the model of an axisymmetric, isotropic, nonlinearly elastic plate that undergoes growth. In particular, I examined whether such a plate can be made to buckle through growth alone.

I discovered that, contrary to my earlier suspicions, for ANY isotropic nonlinearly elastic constitutive relation (stress-strain relation), the deformed but flat configuration is specified by a two-point boundary-value problem, while the buckled configuration is specified by an initial-value problem. Hence the flat and buckled configurations exist simultaneously, as solutions to different classes of problems.

I generalized the form of the constitutive relation to create a class of problems that avoid the messier types of differential algebraic equations. In numerical experimentation with some examples from this class I found that the buckled configuration has lower total elastic strain energy than the corresponding flat configuration.

List publications, if any.

"Spontaneous cavitation in growing elastic membranes" by J. McMahon, A. Goriely, and M. Tabor appeared on the web site of Mathematics and Mechanics of Solids:

http://mms.sagepub.com/cgi/rapidpdf/1081286508092010v1

Check all activities you completed during the funded period:

Academics:

- [x] Independent Study
- [ ] Oral Comprehensive Exam
- [ ] Commence Thesis Research
- [ ] Conference attendance
- [ ] Conference participation
- [ ] Complete PhD

Professional development and outreach:

- [ ] AP Calculus Visit
- [x] High School Workshops
- [ ] Undergraduate Research Project
- [ ] Undergraduate Research Seminar
- [ ] Super TA
- [x] Mentoring junior graduate students for the qualifying exams
- [ ] RTG (help organize)
- [ ] Research Seminar (help organize)

- [ ] Other (please specify)

Attach a brief statment about your academic progress and professional development during the period of support.

Joseph McMahon

VIGRE Funding Report Fall 2008

Part II Report on academic and professional development activities

Write a narrative report on your academic and professional development activities during the period of VIGRE support. This should include a brief description of your progress on dissertation research or, if you have not yet begun dissertation research, a description of your progress toward finding a dissertation advisor and topic and beginning research. You should also discuss any professional development or outreach activities you have undertaken and how they contribute to your career plans.

Academic Progress

During the fall 2008 semester I examined more deeply the model I call the *growing Kirchhoff plate*. This is a nonlinearly elastic plate that undergoes growth and then an elastic response to the growth. In particular, I consider an axially symmetric plate.

Through a study of elementary differential geometry I have gained some mathematical insight into the model. If $\boldsymbol{\chi} : \mathcal{B} \subset \mathbb{E}^3 \to \mathbb{E}^3$ is a diffeomorphism, then it can represent the deformation of a body that occupies the spatial set $\mathcal{B}$. The tensor field $\boldsymbol{F} = \nabla\boldsymbol{\chi}$ is called a *deformation gradient*. If we view the body as a differentiable manifold, then $\boldsymbol{F}$ changes the metric on the manifold. For a true deformation gradient $\boldsymbol{F}$, the new metric is *flat* (as was the original), which means that the manifold can be isometrically embedded in $\mathbb{E}^3$. If we apply a *growth tensor* $\boldsymbol{G}$ that is not the gradient of some diffeomorphism, then the new metric is not flat. Since the body must "live" in $\mathbb{E}^3$, there must be some subsequent change of the metric to produce another flat metric. The goal of the model is to use elasticity theory to find the (elastic) tensor field $\boldsymbol{A}$ such that the tensor field $\boldsymbol{A} \cdot \boldsymbol{G}$ is equal to a deformation gradient.

I have constructed this model to determine whether a nonlinearly elastic plate can be made to buckle by growth alone. In earlier stages I chose a particular constitutive relation (stress-strain relation) and found that, with this relation, a deformed but flat configuration is described by a two-point ODE boundary-value problem, while the buckled configuration is described by a ODE intial-value problem. During the fall semester I realized that this toggling between problem types is found for *all* choices of isotropic constitutive relation. This means that the flat configuration and the buckled configuration of a Kirchhoff plate, which correspond to solutions of different categories of problems, are both accessible to the "grown" plate. In numerical experiments I found that the buckled configuration has lower total elastic strain energy than the corresponding flat configuration.

I also found a class of strain-energy densities that, when inserted into this model, give rise to explicit differential equations, as opposed to the differential algebraic equations that most energies generate. When this is combined with the disovery of the "toggling" between ODE problem types, a much wider variety of conditions can be studied in this model through relatively simple numerics.

Outreach

For the students preparing for January's qualifying exam in applied math, I typed solutions to the August 2008 exam. I made myself available for review sessions, but there was no demand. During the winter break, I provided solutions to several problems via e-mail.

During the fall semester, Bridget Kennedy and I prepared several lessons on probability, statistics, and more basic mathematics for high school students. In early November we presented these ideas and examples to math teachers at Pueblo High School and solicited participation in workshops. We are organizing an event with Jason Dyer, one of the teachers at Pueblo. We plan to hold workshops at the University during this semester.