Print Form

DEPARTMENT OF MATHEMATICS

VIGRE Funding Report

(due 30 days after semester of support)

Semester/Summer and Year: Spring 2008

Name: Benjamin Dyhr

List the graduate courses you have taken this semester (including independent studies), your grades, and the instructors:

Course	Title	Grade	Instructor
MATH920	Dissertation		Tom Kennedy

List the title, date and location of any talks you have given, either here or elsewhere:

"Random Variables Conditioned on Sigma-algebras and Applications in the Sciences", March 6, University of Arizona (Graduate Probability Seminar)

If you are working on your dissertation, include a one paragraph description of your research progress. If you have not yet begun dissertation research, describe your progress toward finding a dissertation topic and advisor and beginning that research.

My dissertation research is in the subject of Schramm Loewner Evolution (SLE), a stochastic differential equation with initial values in the upper half plane that can be used to define a stochastic family of conformal maps. The boundaries of these maps are random fractals related (via limits) to interfaces found in many two dimensional models from mathematical physics. Most SLE research has focused on discrete, probabilistic models exhibiting critical phenomena. This semester I have analyzed a 'skewed' version of SLE thought to be related to 'near-critical' (this has a precise mathematical definition) phenomena of some of the same models. The particular variation of SLE I am interested in comes from substituting a Brownian motion with drift for the standard Brownian motion that defines the SLE stochastic differential equation. The stochastic differential equation is only related to the interface of interest through boundary values of conformal maps, so the bulk of my research involved identifying a limiting process this version of SLE converges to in distribution. My recent dissertation research precisely defines this limiting process, which exhibits stationary increments, via limits of families of conformal maps. The remainder of my dissertation research was focused on proving properties of this limit and investigating near-critical discrete models that I hope to identify or dis-identify with the skewed SLE process.

List publications, if any.

Check all activities you completed during the funded period:

Academics:

- [x] Independent Study
- [ ] Oral Comprehensive Exam
- [ ] Commence Thesis Research
- [x] Conference attendance
- [ ] Conference participation
- [ ] Complete PhD

Professional development and outreach:

- [ ] AP Calculus Visit
- [ ] High School Workshops
- [ ] Undergraduate Research Project
- [ ] Undergraduate Research Seminar
- [x] Super TA
- [ ] Mentoring junior graduate students for the qualifying exams
- [ ] RTG (help organize)
- [x] Research Seminar (help organize)

- [ ] Other (please specify)

Attach a brief statment about your academic progress and professional development during the period of support.

Benjamin Dyhr
VIGRE Report
Spring 2008

Courses Taken: MATH 599: Independent Study (Advisor: Tom Kennedy)

Research:

The SLE_κ process can be defined in any simply connected domain in the complex plane and can be associated with a (random) path $\gamma(t)$ from either (i) an interior point to a boundary point of the domain (radial SLE_κ), or (ii) a boundary point to another boundary point of the domain (chordal SLE_κ). In both cases the SLE_κ process is defined via a solution to a stochastic differential equation called the Loewner differential equation that depends on a Brownian motion with variation κ. In the particular case of chordal SLE_κ in the upper half plane, the initial values of the afforementioned stochastic differential equation are elements of the upper halfplane, but the path $\gamma(t)$ is actually given by boundary behavior of a paramatrized family of conformal maps related to the stochastic inital value problem.

The SLE_κ process was developed to describe scaling limits of two dimensional discrete models from mathematical physics that exhibit critical phenomena, and the theory has been a great success in this regard. My research during the Spring 2008 semester consists of analysis of a 'skewed' version of SLE_κ given by replacing the standard Brownian motion in the SLE_κ Loewner differential by Brownian motion with drift; I will refer to this as 'SLE_κ with drift' from now on. In this adjusted framework it can still be shown that the random path, $\gamma(t)$, exists. However, since the path is only related to the stochastic differential equation through boundary values of conformal maps, identifying interesting properties of $\gamma(t)$ requires technical machinery from complex analysis and theory of stochastic processes. The bulk of my dissertation research during the Spring 2008 VIGRE funding period was concentrated on establishing properties of this path.

Perhaps the most important property of $\gamma(t)$ I proved during the Spring 2008 funding period was the existence of a distributional limit of the process $\gamma^*|_{t>0} = (\gamma(s+t) - \gamma(s))|_{t>0}$ as $s \to \infty$. I have proved that this new path $\gamma^*(t)$ can be extended to $t<0$ and that it exhibits stationary increments. This result used techniques from stochastic and complex analysis and built upon some more basic estimates I had already proved in the past. Also, existence of $\gamma^*(t)$ required defining the process directly using a reverse-flow Loewner equation related to the standard Loewner equation, and the precise mathematical nature of this definition should allow a clear formulation of proofs of many other properties of $\gamma^*(t)$.

The ultimate goal of my dissertation is to extend current SLE_κ theory, which applies to discrete ensembles at criticality, to behaviour of these discrete models *near* criticality (the meaning of 'near critical phenomena' is defined precisely in mathematical physics literature). Very recently, some experts in the field (Wendelin Werner, for example) have written about whether and how this could be done for certain specific discrete models. My hope is to make a precise identification (or dis-identification) of the path $\gamma^*(t)$, from above, with scaling limits of discrete models near criticality. In this regard, my Spring 2008 research has established a firm foundation from which I can build a dissertation by contininuing to investigate these relationships.

In addition to the research outlined above, my Spring 2008 VIGRE funding allowed me time and resources to attend the San Diego AMS meeting from Jan. 6 to Jan. 11, 2008. The research exposure and networking opportunities that were available to me at this huge conference were especially beneficial to my professional development.

Vertical integration activities: I have used the temporal and financial resources provided by my VIGRE grant for the Spring of 2008 to conduct two specific vertical integration activities. I will dedicate a short paragraph to each of these.

First, I contributed my time to lead a one-hour, weekly problem session for graduate students of Dr. Jan Wehr's course in Stochastic Differential Equations (MATH 565c). The purpose of this problem session was (i) to work collectively on specific applications of the theory Dr. Wehr lectured on in class and (ii) to provide time for collective discussion and solution of some of the more difficult homework problems. End of the semester feedback on this problem session from both Dr. Wehr and the students was very postitive.

Second, I organized a semester-long Graduate Probability Seminar composed of a series of weekly speakers for a core group of about six graduate students in our department whose research involves mathematical probability. Several of the speakers talked about applications of mathematical probability in the sciences, and these speakers attracted an even broader audience. Abstracts of all the talks from the seminar (which took place Thursdays at 12:30 pm) can be retrieved from the Arizona Mathematics weekly news archives (www.math.arizona.edu/~weeklynews).