Print Form

DEPARTMENT OF MATHEMATICS

VIGRE Funding Report

(due 30 days after semester of support)

Semester/Summer and Year:
Summer 2008

Name: Benjamin Dyhr

List the graduate courses you have taken this semester (including independent studies), your grades, and the instructors:

Course	Title	Grade	Instructor

List the title, date and location of any talks you have given, either here or elsewhere:

None during funding period.

If you are working on your dissertation, include a one paragraph description of your research progress. If you have not yet begun dissertation research, describe your progress toward finding a dissertation topic and advisor and beginning that research.

After having established the existence of a certain stationary limit of the of a Schramm-Loewner Evolution generating curve driven by a Wiener process endowed with a nonzero drift component in the Spring of 2008, I have dedicated the time and energy allotted by a half-Summer 2008 VIGRE grant to describe and prove exactly the sense in which the limit is stationary with respect to time and space. The time stationarity property of this limit basically follows from the way in which I constructed it. The spacial stationarity property describes an invariance of a distribution of continuous curves in the upper half plane with respect to horizontal shifts of the support of the curve. This latter property requires a more difficult proof, that goes beyond the means used to prove that the limit exists. My summer research was dedicated to outlining and completing specific details of this proof. I have now have non-rigorous means of showing that this property holds and am confident that I can complete a rigorous version of this proof soon. The time and work I have been able to dedicate to this problem during the funding period has put me well in reach of completing my PhD dissertation in the Spring of 2009.

List publications, if any.

Check all activities you completed during the funded period:

Academics:

- [x] Independent Study
- [ ] Oral Comprehensive Exam
- [ ] Commence Thesis Research
- [ ] Conference attendance
- [ ] Conference participation
- [ ] Complete PhD

Professional development and outreach:

- [ ] AP Calculus Visit
- [ ] High School Workshops
- [ ] Undergraduate Research Project
- [ ] Undergraduate Research Seminar
- [ ] Super TA
- [ ] Mentoring junior graduate students for the qualifying exams
- [ ] RTG (help organize)
- [ ] Research Seminar (help organize)

- [ ] Other (please specify)

Attach a brief statment about your academic progress and professional development during the period of support.

Benjamin Dyhr
VIGRE Report
Summer 2008

Research: The SLE_κ process can be defined in any simply connected domain in the complex plane and can be associated with a (random) path $\gamma(t)$ from either (i) an interior point to a boundary point of the domain (radial SLE_κ), or (ii) a boundary point to another boundary point of the domain (chordal SLE_κ). In both cases the SLE_κ process is defined via a solution to a stochastic differential equation called the Loewner differential equation that depends on a Brownian motion with variation κ. In the particular case of chordal SLE_κ in the upper half plane, the initial values of the afforementioned stochastic differential equation are elements of the upper halfplane, but the path $\gamma(t)$ is actually given by boundary behavior of a paramatrized family of conformal maps related to the stochastic inital value problem.

The SLE_κ process was developed to describe scaling limits of two dimensional discrete models from mathematical physics that exhibit critical phenomena, and the theory has been a great success in this regard. My research during the Spring 2008 semester consists of analysis of a 'skewed' version of SLE_κ given by replacing the standard Brownian motion in the SLE_κ Loewner differential by Brownian motion with drift; I will refer to this as 'SLE_κ with drift' from now on. In this adjusted framework it can still be shown that the random path, $\gamma(t)$, exists. However, since the path is only related to the stochastic differential equation through boundary values of conformal maps, identifying interesting properties of $\gamma(t)$ requires technical machinery from complex analysis and theory of stochastic processes.

My VIGRE grant for Summer 2008 was actually one half of a full Summer VIGRE grant typically awarded to graduate students in the University of Arizona VIGRE program, and I used this resource to dedicate myself completely to doctoral dissertation research on SLE_κ with drift during all of July of 2008 and some of August of 2008.

During the Spring 2008 funding period I had established the existence of a distributional limit of the process $\gamma^*|_{t>0} = (\gamma(s+t) - \gamma(s))|_{t>0}$ as $s \to \infty$. I also proved that this new path $\gamma^*(t)$ can be extended to $t<0$. During my Summer 2008 funding period, I continued this research by completing the proof that it exhibits stationary increments. I also used the time to work on a theorem that essentially claims that the distibution of $\gamma^*(t)$ is also invariant under horizontal, spatial shifts of its support. The proof of this Theorem now has been fully outlined and is complete up to some intuitively clear distributional equalities that still need rigorous and precise proof. The time and work I have been able to dedicate to this problem during the funding period has put me well in reach of completing my PhD dissertation in the Spring of 2009.

Vertical integration activities: None during Summer 2008; note that I did several Vertical Integration activities in my Spring 2008 VIGRE funding period directly preceding the summer. These are detailed in my VIGRE report for Spring 2008.