

DEPARTMENT OF MATHEMATICS

VIGRE Funding Report

(due 30 days after semester of support)

Semester/Summer and Year:

Summer 2008

Name: Joseph McMahon

List the graduate courses you have taken this semester (including independent studies), your grades, and the instructors:

Course	Title	Grade	Instructor

List the title, date and location of any talks you have given, either here or elsewhere:

If you are working on your dissertation, include a one paragraph description of your research progress. If you have not yet begun dissertation research, describe your progress toward finding a dissertation topic and advisor and beginning that research.

I continued my research into growing elastic plates. In particular, I continued my development of the axisymmetric growing Kirchhoff disc, which can bend but cannot change thickness. I also began work on a model of the axisymmetric growing Cosserat disc, which can both bend and change thickness. I discovered a peculiarity of my models that, for each disc, causes the problem to change from a two-point boundary-value problem in the case of a flat disc to an initial-value problem if the disc buckles

List publications, if any.

Check all activities you completed during the funded period:

Academics:

- Independent Study
- Oral Comprehensive Exam
- Commence Thesis Research
- Conference attendance
- Conference participation
- Complete PhD

Professional development and outreach:

- AP Calculus Visit
- High School Workshops
- Undergraduate Research Project
- Undergraduate Research Seminar
- Super TA
- Mentoring junior graduate students for the qualifying exams
- RTG (help organize)
- Research Seminar (help organize)

Other (please specify)

Attach a brief statement about your academic progress and professional development during the period of support.

Joseph McMahon

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Part II Report on academic and professional development activities

Write a narrative report on your academic and professional development activities during the period of VIGRE support. This should include a brief description of your progress on dissertation research or, if you have not yet begun dissertation research, a description of your progress toward finding a dissertation advisor and topic and beginning research. You should also discuss any professional development or outreach activities you have undertaken and how they contribute to your career plans.

Academic Progress

I continued my research into reduced models of growing elastic plates. Having derived ordinary differential equations for the static elastic deformation of an axisymmetric, anelastically “grown” plate that can bend but cannot change its thickness, I found an analytical solution for the flat configuration and produced some numerical results describing a slightly bent configuration. This buckling is caused by the anelastic growth alone; there are no applied loads in this model or the next.

I derived equations for a plate that is similar, except that it can change its thickness in its elastic response to the anelastic growth. This model, which I call the *Cosserat* disc as opposed to the earlier *Kirchhoff* disc, has proved far messier. The ordinary differential equations feature singular points at the origin, and one of them has a singular perturbation. Weeks of work in Mathematica have yielded no success in integrating the equations numerically. Thanks to the singular perturbation, even sixteen digits of accuracy in the initial condition are not enough to avoid explosive growth of a quantity that should remain on the order of 1 to 10.

In the midst of this numerical work, I realized that the neo-Hookean strain-stress relation I chose for these models induces a curious and convenient change in type of problem if the disc buckles. If the disc’s elastic deformation leaves the disc flat, the problem is a two-point boundary-value problem; zero radial traction is the boundary condition that must be satisfied at the periphery of the disc. On the other hand, if the disc buckles, all radial and azimuthal stress vanishes, and the problem becomes an initial-value problem. I’ve shown that this is an artifact of the choice of the neo-Hookean strain-stress relation. If \mathbf{A} is the elastic deformation tensor, then isotropic hyperelastic constitutive relations can be considered functions of the symmetric tensor $\mathbf{A}\mathbf{A}^T$. The key to the switch of problem type is the simple dependence of the neo-Hookean relation on the first principal invariant (the trace) of $\mathbf{A}\mathbf{A}^T$.

Outreach

Starting in late May, I met weekly with the students preparing for August's qualifying exam in applied mathematics. After a rapid review of some fundamental aspects of linear algebra and matrix theory, the students and I worked through old exams. Except for one week, we met each Friday morning until the end of July. In May the students seemed to view this preparation as a daunting task, but by the final weeks they were solving almost all problems without any assistance from me.

Some of the solutions I wrote can be found on my departmental web page, at

math.arizona.edu/~jmcmahon

under the heading *Some notes I've written*. By the end of the summer, I was still learning far more per week than the students were, as I had forgotten a vast amount since I had taken the exam. Each time I assist with this summer preparation, my skills improve. For example, this summer I found significant errors in several of the solutions I wrote as a tutor during the summer of 2007. Further, Professor Venkataramani's questions have a flavor different from those I've seen in earlier exams.

Professional Development

During my research this summer I developed some skill with Mathematica's command `NDSolve`, which is for numerical integration of systems of differential equations. I also started learning the rudiments of FORTRAN, in preparation for a problem that, it turns out, may be amenable to Mathematica instead.

I spent some time preparing a *curriculum vitae* for my departmental web page. At the time I write this the *c.v.* is incomplete, but it now includes all my publications (except for one whose publication date has not been announced), a history of my financial support for academics, a list of my presentations, and a history of my research experience.