Print Form

DEPARTMENT OF MATHEMATICS

VIGRE Funding Report

(due 30 days after semester of support)

Semester/Summer and Year: Summer 2008

Name: Jordan Schettler

List the graduate courses you have taken this semester (including independent studies), your grades, and the instructors:

Course	Title	Grade	Instructor

List the title, date and location of any talks you have given, either here or elsewhere:

If you are working on your dissertation, include a one paragraph description of your research progress. If you have not yet begun dissertation research, describe your progress toward finding a dissertation topic and advisor and beginning that research.

I will be starting the written component of my comprehensive exam in the fall, and have done background reading in Iwasawa theory to that end. My reading has entailed an advanced monograph posted on Ralph Greenberg's website (orignally suggested to me by Prof. Kirti Joshi) supplemented by the class notes of Edray Goins on class field theory and J. S. Milne on algebraic number theory (which together consituted the course texts for the sequence in number theory I took last year). I will be working with Prof. William McCallum to continue this reading with the goal of completing my comprehensive requirements by the end of the fall semester. Prof. McCallum would make an excellent thesis advisor as would Prof. Joshi (with whom I did my RTG and with whom I'll be working with as a super TA next semester) or Prof. Thakur, both of whom I intend upon having on my orals committee.

List publications, if any.

Check all activities you completed during the funded period:

Academics:

- [x] Independent Study
- [ ] Oral Comprehensive Exam
- [ ] Commence Thesis Research
- [ ] Conference attendance
- [ ] Conference participation
- [ ] Complete PhD

Professional development and outreach:

- [ ] AP Calculus Visit
- [ ] High School Workshops
- [ ] Undergraduate Research Project
- [ ] Undergraduate Research Seminar
- [ ] Super TA
- [x] Mentoring junior graduate students for the qualifying exams
- [ ] RTG (help organize)
- [ ] Research Seminar (help organize)

- [x] Other (please specify)

Completed language exam in German by translating an article on Goldbach's conjecture. Participated as a senior graduate student in the 2008 integration workshop. Wrote a new project for the 2008 integration workshop on Eisenstein's proof of quadratic reciprocity.

Attach a brief statment about your academic progress and professional development during the period of support.

VIGRE FUNDING REPORT PART II

JORDAN SCHETTLER

Academic and professional development activities

1. Research

As intended, I began reading Greenberg's monograph on Iwasawa theory and have filled in most of the details from the first few sections which cover the inspirations for Iwasawa theory in the case of a single extension of number fields. In the process, I have also reviewed some class notes of Milne on algebraic number theory and class notes of Goins on class field theory (which together constituted the texts for the year long sequence in number theory I took last year). Also, I'm in the process of reading an article by Iwasawa himself on Kida's generalization of the Riemann-Hurwitz formula to number fields. The content of this article together with understanding the proof of Iwasawa's growth formula (as presented in Greenberg) should constitute a healthy amount of material for my comprehensives.

I'll sketch some of the major results from the sections I've read so far from Greenberg. Let F'/F be a finite extension of number fields and let H'/H be the corresponding extension of Hilbert class fields. Then the norm map $N_{F'/F}$ on the ideal class groups is surjective when $H \cap F' = F$; likewise, the restriction to the p-parts of the class groups is surjective when $L \cap F' = F$ where L is the p-Hilbert class field. If the extension F'/F happens to be cyclic and at most one prime of F is ramified in F', then we can also determine the kernel of $N_{F'/F}$ as $\ker(N_{F'/F}) = Cl_{F'}^{\sigma-1}$ where σ is any generator of the Galois group $G = \mathrm{Gal}(F'/F)$. If, in addition, G is a p-group and there's a prime of F which is totally ramified in F', then $p|h_{F'} \Leftrightarrow p|h_F$.

Now let $J_{F'/F} : Cl_F \to Cl_{F'}$ be given by sending an ideal class c to the ideal class of $I\mathcal{O}_{F'}$ where I is an ideal in c. If F'/F is cyclic of degree p, then using the action of G on the unit group $\mathcal{O}_{F'}^{\times}$ and a little Galois cohomology we can bound the size of the kernel by $|\ker(J_{F'/F})| \leq p^{r+2}$ where r is the rank of $\mathcal{O}_{F'}^{\times}$. If F'/F is nontrivial cyclic and unramified, then $\ker(J_{F'/F}) \cong H^1(F'/F, \mathcal{O}_{F'}^{\times})$ is nontrivial (which is Hilbert's theorem 94). In addition, I read through several examples for which the kernel is nontrivial in ramified extensions.

2. Professional Development

This summer I ran a problem session for analysis to help first year students prepare for the qualifying exam in that subject. We met twice a week (Tuesdays and Fridays) for three hours each (10:00 am to 1:00 pm). For the first two and one half week's we ran through the Arizona qual packet. For the remaining weeks I scoured through quals from Texas, UCLA, Iowa State, LSU, Georgia, and Penn State (as well as old Arizona quals not included in the current packet) to cook up 7 to 8 qual-like problems per session. I would write the problems on the blackboard at 10:00 am, and the students would then spend two hours working on these problems asking for clarifications or hints along the way. At noon I'd start asking for volunteers to present their solutions to each problem one by one. If no one had a solution to a given problem, then I would either put up my solution or leave the problem for the next session. I also answered questions via email and met several students outside the session for extra help at their request. This experience helped prepare me for what I might expect running the algebra sessions as a super TA next semester. In addition, it strengthened my own knowledge in this core subject (I'm of the opinion that being able to explain the main ideas of a subject at a graduate level is the true mark of having a solid grip on the material).

I also participated in the 2008 Integration Workshop. I gave suggestions on how to work through some questions in problem sets and helped students better explore their own ideas on these problems. I wrote a project for this Integration Workshop on Eisenstein's proof of quadratic reciprocity (the pdf of which is available on the math website). One group happened to choose this project, so I was able to answer any questions they had about it. Working at the workshop again gave me the opportunity to meet the incoming grad students and improve my interaction skills.