Print Form

DEPARTMENT OF MATHEMATICS

VIGRE Funding Report

(due 30 days after semester of support)

Semester/Summer and Year: Summer 2008

Name: Jefferson Taft

List the graduate courses you have taken this semester (including independent studies), your grades, and the instructors:

Course	Title	Grade	Instructor

List the title, date and location of any talks you have given, either here or elsewhere:

If you are working on your dissertation, include a one paragraph description of your research progress. If you have not yet begun dissertation research, describe your progress toward finding a dissertation topic and advisor and beginning that research.

I have demonstrated that the level set method used to analyze singularities of the mean curvature flow will not readily generalize to Ricci flow on hypersurfaces of Euclidean four-space. If there is a level set method in this situation, it will need to have a different formulation, and would probably only work in very special cases. I have also obtained unrelated results about which I will write two papers to be submitted for publication during the Fall semester. One paper will be about the total Gauss curvature of three-manifolds immersed in Euclidean four-space, and the other will be about Cross-Curvature flow on space-like hypersurfaces of Minkowski four-space.

List publications, if any.

Check all activities you completed during the funded period:

Academics:

- [x] Independent Study
- [ ] Oral Comprehensive Exam
- [ ] Commence Thesis Research
- [x] Conference attendance
- [ ] Conference participation
- [ ] Complete PhD

Professional development and outreach:

- [ ] AP Calculus Visit
- [ ] High School Workshops
- [x] Undergraduate Research Project
- [ ] Undergraduate Research Seminar
- [ ] Super TA
- [ ] Mentoring junior graduate students for the qualifying exams
- [ ] RTG (help organize)
- [ ] Research Seminar (help organize)

- [ ] Other (please specify)

Attach a brief statment about your academic progress and professional development during the period of support.

Proposed Plan of Study and Proposed Vertical Integration Activities

Jefferson Taft

Sept. 12, 2008

This summer I set out to analyze neckpinch singularites of the Ricci-flow using a level set method like the one used in mean curvature flow. What I found instead was a very convincing counter-example illustrating why such a thing could not exist. In particular, the main point of the level set method in mean curvature is that two manifolds which are embedded in Euclidean space and not touching at initial time, will never touch at any later time. I can now demonstrate an example of two three-manifolds embedded in $\mathbb{R}^4$ which do not touch at initial time, but touch a short time later.

I also discovered this summer that in the case of certain rotationally symmetric manifolds, the Ricci flow can be formulated as a parabolic PDE instead of a system of PDE's. Should a weak solution to this PDE be found, it would provide an example of a Ricci flow that 'flows through' a neckpinch singularity.

Inspired by this idea of embedding manifolds with some kind of symmetry and reformulating geometric flows as a PDE, I did a similar thing with what is called the cross-curvature flow. The cross-curvature flow is a geometric flow that seems to prefer negatively curved manifolds in the same way that the Ricci flow prefers positively curved manifolds. Compared to the Ricci flow, relatively little is known about the cross-curvature flow. Only a few papers exist about it, and as of now, the literature is more focused on conjectures than theorems. We can embed a three-manifold homotopic to $\mathbb{R}^3$ into Minkowski four-space and induce a negatively curved Riemannian metric on it. Then, assuming certain symmetries, the cross curvature flow can be formulated as a parabolic PDE with no reaction term. Depending on what the analysis of this PDE yields, a paper will be submitted for publication about this.

Another project this summer was to develop an observation about the total Gauss curvature of immersed three dimensional submanifolds of $\mathbb{R}^4$. Namely, it seems that this functional is preserved under arbitrary deformations. I first made this observation on hypersurfaces of revolution, then produced a proof for the more general case. Interestingly, we have a counterexample to demonstrate that this is not true for a manifold which is not locally embeddable into $\mathbb{R}^4$. We are still in the process of conducting a massive literature search in order to find out whether this is known or not, and if it is known, whether I have the same proof or a different one. This result would imply, among other things, that the three-sphere is not evertible (able to be turned inside out) in the same way that the two-sphere is. Should either the theorem or the proof be unknown (and correct), there will be a paper submitted for publication.

During the summer I had the priveledge of attending a conference entitled "Ricci Flow and Related Topics". This conference was held at the Henri Poincare Institute in Paris. The conference featured many active researchers in Ricci Flow as well as geometric flows in general. It gave me an opportunity not only to hear about active research directions, but also to meet some expert geometers.

My vertical integration activity was helping my advisor, Dr. Glickenstein, with his geometry lab. Ultimately, the goal is to provide visualizations to help people understand geometry. This summer, we developed software that simulated discrete versions of the Ricci flow and software that searches for 'weighted delauney triangulations'. The undergraduates did all the coding, with myself and two other graduate students acting as mathematical consultants. Basically, we helped them understand the math behind the software. One of our undergraduates, Alex, eventually proposed a renormalization of one of these flows, which seems to work better numerically than other versions. Dr. Glickenstein has asked his graduate students to work on some of the related calculations, and if the math agrees with the experimental data, this flow might be included in an upcoming paper.