Print Form

DEPARTMENT OF MATHEMATICS

VIGRE Funding Report

(due 30 days after semester of support)

Semester/Summer and Year: Spring 2009

Name: Joseph McMahon

List the graduate courses you have taken this semester (including independent studies), your grades, and the instructors:

Course	Title	Grade	Instructor
MATH 920	Dissertation	S	Goriely

List the title, date and location of any talks you have given, either here or elsewhere:

23 January 2009: Quaternions for Fun and Profit, Brown Bag Colloquium
Program in Applied Mathematics, The University of Arizona

If you are working on your dissertation, include a one paragraph description of your research progress. If you have not yet begun dissertation research, describe your progress toward finding a dissertation topic and advisor and beginning that research.

At the beginning of the semester I discovered an error in my derivation of equations describing plates that have buckled due solely to "incompatible" growth. I derived new and correct equations. An interesting result I discovered in fall 2008 does not hold for the correct equations.

I also derived equations for the related problem of an incompatibly grown but unbuckled plate. I proved the existence of solutions and found numerical solutions for a variety of growth types. These results have been written up and are being prepared for submission for publication.

I solidified my understanding of the differential geometry underlying incompatible growth of a solid body and used it to offer an interpretation of such growth that differs from the one most popular in elasticity literature.

List publications, if any.

Check all activities you completed during the funded period:

Academics:

- [x] Independent Study
- [ ] Oral Comprehensive Exam
- [ ] Commence Thesis Research
- [x] Conference attendance
- [ ] Conference participation
- [ ] Complete PhD

Professional development and outreach:

- [ ] AP Calculus Visit
- [ ] High School Workshops
- [ ] Undergraduate Research Project
- [ ] Undergraduate Research Seminar
- [x] Super TA
- [ ] Mentoring junior graduate students for the qualifying exams
- [ ] RTG (help organize)
- [ ] Research Seminar (help organize)

- [ ] Other (please specify)

Attach a brief statment about your academic progress and professional development during the period of support.

I. Academics

Having worked for months on a model of the buckling of a thin axisymmetric plate that has undergone "incompatible" growth and having found a rather nice result showing that the buckled plate is decsribed by an initial-value problem while the unbuckled case is described by a two-point boundary-value problem, I found in the spring 2009 semester that I had made an error during the derivation of the differential equations for this problem. After deriving the equations correctly, I found that the dichotomy between the types of the problems is absent; each is, in fact, a two-point boundary-value problem.

With the correct equations for the buckled plate in hand, I sought solutions via numerical integration. After a variety of attempts failed, I turned to the equations for the unbuckled configuration. I used a change-of-variables to convert the original equations into a set of autonomous equations. For particular choices of incompatible growth, I used some basic results of dynamical systems theory to prove the existence of solutions of the problem.

I produced numerical solutions with a method that exploited the form of the existence proof. I'm considering unloaded plates, so the stress found in the results arises solely from the body's elastic response to incompatible growth. In some examples the residual stress is compressive through most of the body, which suggests that the body may be prone to buckling. The growth tensors used in these examples will be the first used in the search for buckled configurations, which correspond to a very different set of euations.

I have also come to understand incompatible growth via elementary differential geometry. In the description used in much of the literature I've read, incompatible growth is viewed as expanding and/or contracting infinitesimal volumes in such a way that they cannot form a continuous body. The elastic response of the body deforms these stress-free volumes so that they form a body again. After this elastic response, the body carries stress, even in the absence of loading and of body forces.

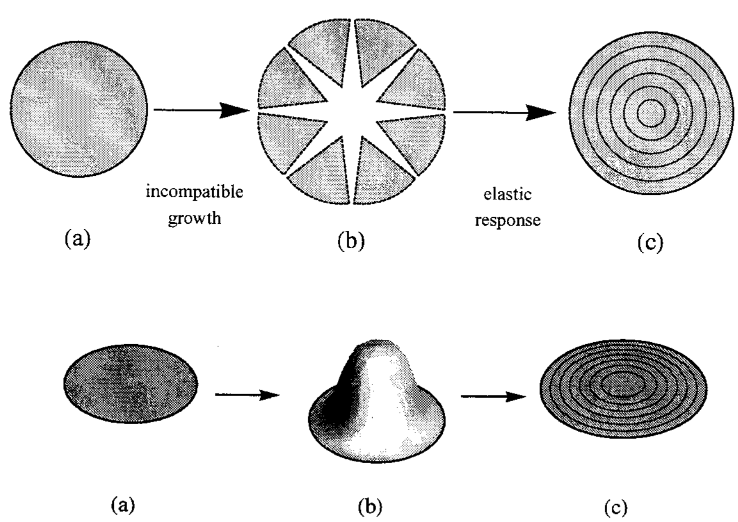

Figure 1: Incompatible growth is often seen as creating a "virtual configuration" consisting of stress-free volumes that cannot form a continuous body (top). The body's elastic response contorts these volumes in such a way that they again form a continuous body but carry stress even in the absence of loading and body forces. However, viewing the body as a Riemannian manifold (bottom) allows us to keep continuity of the body at all times, so we needn't consider the energetics of tearing and re-attaching volumes. After incompatible growth, the body does not fit into Euclidean space (b). The elastic response of the body changes the metric tensor so that the body can be isometrically immersed in $\mathbb{E}^3$, as it was before growth.

If we view the body as a Riemannian manifold, however, we get a more convenient interpretation. Originally, the body has a metric tensor that defines distances between points in the space of coordinates respresenting material points of the body. Before growth, the distances between coordinate points agree with the distances between the Euclidean images of the coordinate points. Incompatible growth changes the metric tensor to one that is not isometrically immersible in $\mathbb{E}^3$. In other words, the distances between coordinate points do not match the collection of distances of any Euclidean image of the body. Since the body must remain in $\mathbb{E}^3$, the elastic response changes the metric tensor once again, to one that allows the body to be isometrically immersed in $\mathbb{E}^3$. Through the whole process, the body remains a manifold, so there are no issues of creating and annihilating interfaces between volumes of matter.

II. Professional Development and Outreach

On January 24, I attended the Mathematics Educator Appreciation Day (MEAD) Conference, held at Tucson High Magnet School.

I served as the super-TA for Math 583-B, the second semester of Principles of Applied Mathematics. In weekly meetings I gave insight and intuition into the topics covered during the semester, inclulding the theory of distributions and tempered distribtions, linear differential and integral operators, spectral theory of linear operators, calculus of variations, and perturbation theory.

In most cases, I tried to attach intuition derived from more basic mathematics. For example, much of the spectral theory of linear operators in Hilbert space can be viewed as an extension of the spectral theory of matrices. And calculus of variations may seem more familiar when the first variation of a functional is viewed as a directional derivative in an infinite-dimensional Hilbert space. I also adopted numerous examples from old qualifying exams, with an eye toward letting the students know the variety of problems that may appear on that exam in August.