Print Form

DEPARTMENT OF MATHEMATICS

VIGRE Funding Report

(due 30 days after semester of support)

Semester/Summer and Year: Spring 2009

Name: Benjamin Pittman-Polletta

List the graduate courses you have taken this semester (including independent studies), your grades, and the instructors:

Course	Title	Grade	Instructor

List the title, date and location of any talks you have given, either here or elsewhere:

If you are working on your dissertation, include a one paragraph description of your research progress. If you have not yet begun dissertation research, describe your progress toward finding a dissertation topic and advisor and beginning that research.

I finished a paper with my advisor, Doug Pickrell, entitled 'Unitary loop groups and factorization', which was submitted to 'Journal of Lie Theory'. Two of the results from this paper, obtained over the summer, will go into my dissertation.

List publications, if any.

'Unitary loop groups and factorization', arXiv:0905.2911v2 (with Doug Pickrell), 2009.

Check all activities you completed during the funded period:

Academics:

- ☐ Independent Study
- ☐ Oral Comprehensive Exam
- ☒ Commence Thesis Research
- ☐ Conference attendance
- ☐ Conference participation
- ☐ Complete PhD

Professional development and outreach:

- ☐ AP Calculus Visit
- ☐ High School Workshops
- ☐ Undergraduate Research Project
- ☐ Undergraduate Research Seminar
- ☐ Super TA
- ☒ Mentoring junior graduate students for the qualifying exams
- ☐ RTG (help organize)
- ☐ Research Seminar (help organize)

☒ Other (please specify)

I drafted and implemented a proposal for Class-to-Class Mentoring with Suz Tolwinski.

Attach a brief statment about your academic progress and professional development during the period of support.

SUMMER 2009 VIGRE REPORT, PART II

BENJAMIN PITTMAN-POLLETTA

This past summer was very productive. I made significant research progress, traveled to two conferences, and accomplished two vertical integration projects.

1. Research

With my advisor, Doug Pickrell, I recently completed a paper entitled 'Unitary loop groups and factorization', which has been submitted to *Journal of Lie Theory* [4]. In this paper, we discuss a refinement of triangular factorization for the loop group of an arbitrary compact group K, denoted LK. (Please see my VIGRE proposal for details.)

I completed two of the results appearing in this paper over the summer, both of which were goals mentioned in my VIGRE proposal. First, I provided a constructive proof of the characterization of loops having a triangular factorization

$$k(z) = l(z)au(z),$$

where $l(z)$ is both antiholomorphic and unipotent upper-triangular, a is diagonal and real, and $u(z)$ is holomorphic, and unipotent upper triangular at zero. The characterization of such loops is the following: given a representation π of K with highest weight vector v, the vector $\pi\left(k^{-1}\right)v$ must be holomorphic, and a real multiple of v at zero. (For $k \in LSU(n)$, if k^i is the i^{th} row of k, this means that for $2 \leq i \leq n$,

$$k^i \wedge \ldots \wedge k^n$$

is a holomorphic map into $\wedge^{n-i+1}\mathbb{C}^n$ whose value at zero is a real multiple of $e_i \wedge \ldots \wedge e_n$.) This condition is exactly that needed to perform a pointwise triangular factorization of k near zero, and from this pointwise triangular factorization can then be constructed the triangular factorization we seek.

Second, I was able to prove the equivalence of the existence of a triangular factorization $k = l(z)mau(z)$ and the existence of a factorization

$$k = k_1^*(z)\lambda(z)k_2(z),$$

where $\lambda(z)$ is diagonal and unitary, and the unitary loops $k_i(z)$ have triangular factorizations

$$k_1(z) = l_1(z)a_1u_1(z), \qquad k_2(z) = l_2(z)a_2u_2(z),$$

with a_i diagonal and real, $u_i(z)$ holomorphic and unipotent upper-triangular at zero, $l_i(z)$ antiholomorphic, $l_1(z)$ unipotent lower-triangular, and $l_2(z)$ unipotent upper-triangular, for arbitrary K. This was accomplished using the Iwasawa decomposition to rewrite a previous argument, relying on a modification of the Gram-Schmidt algorithm, in a representation-independent way.

Currently, I am investigating several questions related to this paper, in two areas. The first concerns minimal periodic sequences in the affine Weyl group associated to a loop group. Our refinement of triangular factorization depends on reduced decomposition of an element of the affine Weyl group. Is there a canonical such element, and is there a canonical decomposition of this element? Given an element in the affine Weyl group, how many reduced decompositions does it have? How does the choice of a Weyl group element and a reduced decomposition affect the resulting factorization?

Secondly, I am exploring the relationship of this factorization to Poisson geometry. In finite dimensions, the analogous factorization provides coordinates on the Birkhoff cells of a semisimple Lie group. If the Lie group is seen as a symmetric space, it inherits a canonical Poisson structure, called the Evens-Lu Poisson structure [2]. The coordinates given by the factorization then diagonalize this Poisson structure [1],[3]. There should be a similar relationship in infinite dimensions.

Date: September, 2009.

2. Professional Development

Late in the summer, I attended two conferences. From July 27 through August 1, I attended the 33rd Conference on Stochastic Processes and Their Applications, in Berlin. From August 3 through August 10, I attended the XVI International Congress on Mathematical Physics in Prague. I found both conferences to be useful. I saw a wide variety of interesting talks, and met a number of graduate students and young researchers in probability and mathematical physics.

3. Vertical Integration

During the summer, as outlined in my VIGRE proposal, I ran review sessions to help the current second-year graduate students prepare for their qualifying exams. The review sessions went well, as evidenced by the high rate of success of the students.

I also drafted and implemented a proposal for Class-to-Class Mentoring with Suz Tolwinski, the current graduate student representative in the IPAM. Class-to-Class Mentoring augments the advising structure in the department, and strengthens the IPAM community, by pairing beginning graduate students with more advanced graduate student mentors on a class-to-class, rather than individual-to-individual, basis. The first Class-to-Class Mentoring meetings were held on August 21st, and were a great success. The students who participated asked us to arrange further meetings during the semester, and Suz and I are currently working on this.

References

[1] Caine A. & Pickrell D. "Homogeneous Poisson structures on symmetric spaces." *International Mathematics Research Notices* vol. 2009 no. 1. 2008.

[2] Evens S. & Lu J. "On the variety of Lagrangian subalgebras, I." *Annales Scientifique de LÉcole Normale Supérieure* 34. 2001.

[3] Lu J. "Coordinates on Schubert cells, Kostant's harmonic forms, and the Bruhat-Poisson structure on G/B." *Transformation Groups* v. 4 no. 4. 1999.

[4] Pickrell D. & Pittman-Polletta B. "Unitary loop groups and factorization." arXiv:0905.2911v2. 2009.