What every incoming student is expected to know

Here is a list of topics from various areas of mathematics which we expect that incoming students know. Of course, few if any students will have a complete command of all of these topics, and so we are making the list available as an aid in preparing to enter the program. We strongly encourage each entering student to review (or learn) this material, using the references mentioned or other similar ones.

We note that the phrase "know" in the first sentence above does not mean "have seen." Rather, we expect that students will know the definitions, the main examples, and that he or she can solve interesting problems using these concepts. The best way to test your level of understanding is to get a book on the subject and go directly to the exercises. If you can do most of them without breaking a sweat, then you "know" the topic.

Linear Algebra

1. Vector spaces and linear transformations. Various subspaces and quotient spaces associated with a linear transformation.
2. Bases in a linear space. The matrix of a linear transformation with respect to two bases. Change of basis formula.
3. Gaussian elimination ("row reduction") and applications to systems of equations, ranks.
4. The dual space and the adjoint of a linear transformation.
5. Determinants, traces, characteristic and minimal polynomials.
6. Eigenvectors and eigenvalues. Generalized eigenvectors. The Jordan form. The determinant, trace, and characteristic polynomial in terms of the eigenvalues.
7. Scalar products (both complex and real). Matrix of a scalar product and change of basis formula. Gramm-Schmidt process.
8. Self-adjoint and symmetric operators. Unitary and orthogonal operators. Spectral theorems.

References: There are many but Strang, "Linear Algebra and Its Applications" is a classic. An interesting recent book is Axler, "Linear Algebra Done Right." See also Artin, "Algebra."

Vector Calculus and Differential Equations

1. The derivative of a function $f : \mathbf{R}^n \to \mathbf{R}^m$.
2. Line integrals. The multidimensional Riemann integral. Change of variables formula.
3. Div, grad, curl. Divergence theorem, Stokes' theorem, Green's theorem.
4. Existence and uniqueness of solutions to a first order ODE.
5. First and second order linear ODEs with constant coefficients.
6. Higher order linear ODEs and systems of first order ODEs.

References: Again there are many. We suggest Rudin, "Principles of Mathematical Analysis" for Calculus, Boyce and DiPrima "Elementary Differential Equations" for ODEs, and Courant "Differential and Integral Calculus" for both.

Complex Analysis

1. Holomorphic (analytic) functions.
2. Elementary functions and the mappings that they define.

3. Conformal mappings. Problems such as finding a conformal mapping from a quadrant to a disc.
4. The Taylor series of a holomorphic function.
5. Contour integrals. Cauchy's theorem. The Cauchy formula.
6. Poles. Meromorphic functions. Laurent series. Residues.
7. Evaluating contour integrals of meromorphic functions.
8. Using complex integration for evaluating some improper integrals.

References: Churchhill and Brown "Complex Variables and Applications," Ahlfors "Complex Analysis," or Conway "Functions of One Complex Variable."

Algebra

1. Groups and homomorphisms of groups. Basic examples: permutation groups, alternating groups, cyclic groups, dihedral groups, linear groups.
2. Subgroups, normal subgroups, quotient groups, product groups.
3. Group actions on sets. Orbits and their properties.
4. Structure of finitely generated abelian groups.
5. Rings, ideals, quotient rings.
6. Prime ideal and maximal ideals.
7. Principal ideals and principal ideal domains (PIDs). Examples of PIDs (integers, polynomial rings). Division algorithm and factorization in PIDs.
8. Modules. Structure of finitely generated modules over a PID.

References: Artin "Algebra," or Dummitt and Foote "Abstract Algebra." If these are too advanced, try Jacobson, Herstein, Fraleigh, Gallian, Hungerford, ...

Real Analysis

1. Improper integrals. Convergence tests.
2. Sequences and series. Taylor series. Radius of convergence.
3. Series of functions. Uniform convergence.
4. Exchanging the order of summation and limit.
5. Differentiation and integration of series of functions.
6. Countable and uncountable sets. Axiom of choice and Zorn's lemma.
7. Axioms for $\mathbf{R}$

References: Rudin and Courant (see Calculus above). Spivak "Calculus."

Point set topology (Don't panic ... although many of you will not have had a course in this, much of the needed material is covered in advanced calculus or real analysis courses.)

1. Open, closed, connected, and compact subsets of $\mathbf{R}^n$. Heine-Borel theorem.
2. Topological spaces and Hausdorff spaces.
3. Metric spaces as topological spaces.
4. Continuous functions.
5. Compact sets. Images of compact sets are compact. Continuous functions achieve their maximum on compact sets.
6. Connected sets. Images of connected sets are connected.

References: Munkres "Topology: A First Course" or Singer and Thorpe "Lecture Notes on Topology and Geometry." See also your real analysis references.