

Integration Workshop 2006
Project descriptions

1. **Generating functions:** How to use generating functions to solve linear recurrences.
2. **Nice groups actions on the upper half plane:** The topology and geometry of actions of suitable groups of matrices on the upper half plane.
3. **The space of circles:** Some geometry of the set of all circles in the plane.
4. **The space of metric spaces:** The set of all compact metric spaces can itself be made into a metric space.
5. **Constructing Brownian motion:** Basics on the Wiener process. (Requires somewhat sophisticated analysis.)
6. **Solving elliptic PDEs:** Hilbert space ideas can be used to solve elliptic PDEs.
7. **Lie brackets and parking a car:** The dynamical system associated to a vector field, with applications to control theory.
8. **Stone-Weierstrass:** Generalizes the fact that a continuous function on a finite closed interval can be uniformly approximated by polynomials.
9. **Gelfand-Mazur:** Every element of a complex Banach algebra has a non-empty, compact spectrum.
10. **Topological groups and duality:** Basics plus the result that Pontryagin duality interchanges compact and discrete topological groups.
11. **Urysohn metrization theorem:** Characterizes those topological spaces which admit a metric inducing the given topology.
12. **Linear differential equations and the Jordan form:** Knowledge of the Jordan form of a linear transformation leads quickly to a characterization of the solutions of a linear, constant-coefficient differential equation.
13. **Combinatorial topology of dynamical systems:** How the topology of a surface affects the fixed points of the flow of a vector field.
14. **Geometric aspects of elliptic curves:** Some basics on the complex plane modulo a lattice as a space and as a group.
15. **Group representations:** Basics on groups acting on vector spaces.
16. **Topological vector spaces:** A finite dimensional real vector space has a unique reasonable topology.
17. **The Weierstrass \wp -function:** Basics on doubly periodic functions on the complex plane.
18. **Harmonic analysis and the distribution of numbers:** Ideas of Fourier analysis leading to Weyl's theorem that the multiples of an irrational angle are equidistributed in the circle.
19. **Constructing the p -adic numbers:** The p -adics are a completion of the rationals in the same sense that the reals are a completion of the rationals. The project gives two constructions and shows that they yield the same object (a topological field).
20. **Modules over PIDs:** A proof of the fundamental structure theorem for said modules.
21. **Winding numbers:** How many times does a closed curve in the plane wind around a point? Making this precise has amazing applications like the Brouwer fixed point theorem and the fundamental theorem of algebra.
22. **Primes in an arithmetic progression:** Uses analytic functions to show that if a is relatively prime to m , then the sequence $a, a + m, a + 2m, \dots$ contains infinitely many prime numbers.
23. **Compact Hausdorff spaces and C^* algebras:** A compact space determines and is determined by its algebra of functions.
24. **Calculating rational canonical forms:** How to find a nice standard form (different than the Jordan form) of a matrix.