

INTEGRATION WORKSHOP, AUGUST 4–8, 2006

CALCULUS

1. LECTURE 1: MULTIVARIABLE CALCULUS:

1.1. Differentiation.

- Derivative, local linearization Jacobian, chain rule
- Directional derivative, gradient in terms of partial derivatives
- Mean value theorem
- Higher derivatives, multi-dimensional Taylor's formula.
- Implicit functions, "solving" multidimensional equations and inverse function theorem
- Optimization and Lagrange multipliers.

1.2. Integration.

- Multidimensional Riemann integrals, evaluation as an iterated integral.
- Vector fields on \mathbb{R}^n , associated differential operators and ODEs.
- div, grad and curl. Fundamental theorem of calculus, Stokes', Green's and Gauss' (divergence) theorems.

2. LECTURE 2: ANALYSIS

2.1. Basic notions.

- Metric spaces, convergence of sequences and topology.
- Norms on vector spaces.

2.2. Analysis on \mathbb{R} .

- Completeness of \mathbb{R} , least upper bound property.
- sequences and series, liminf and limsup, limit points, convergence tests, absolute and conditional convergence, rearrangements of series.
- countable and uncountable sets,
- Separability of \mathbb{R} , analysis through approximation by rationals
- Open and closed sets in \mathbb{R} , the Borel sigma algebra

2.3. Analysis of real valued functions.

- real valued functions, upper and lower envelopes, semicontinuity and continuity.
- sequences and series of functions.
- Pointwise and uniform convergence, L^∞ norm and completeness of $C([0, 1])$.
- When can we interchange the order of limiting processes? Differentiation and integration of a series of functions.
- Stone-Weierstrass theorem and approximation of continuous functions by "rational" polynomials through uniform convergence in $C([0, 1])$.

3. LECTURE 3: SOME APPLICATIONS

3.1. Complex analysis.

- Power series, radius of convergence.
- smooth non-analytic real valued functions.
- Analytic (holomorphic) functions, Cauchy-Riemann equations and conformal mappings.
- Contour integrals, Cauchy's theorem and the Cauchy formula, comparison with Taylor's formula.
- Meromorphic functions and poles, Laurent series.
- Evaluating contour integrals using residues, applications.

3.2. Differential equations.

- First order ODEs = Dynamical systems, initial value problem
- Autonomous and non-autonomous systems
- Linear systems, fundamental matrix
- Gronwall's inequality, uniqueness of solutions
- Existence of solutions (Picard iteration)
- Phase plane analysis
- fixed points and stability.