INTEGRATION WORKSHOP 2007: LINEAR ALGEBRA LECTURE PLAN

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The following are outlines for the three linear algebra lectures for the Integration Workshop. There are many textbooks which are useful resources for this material, but the following I find especially useful:

Sheldon Axler, Linear Algebra Done Right. Second Edition. Undergraduate Texts in Mathematics. *Springer–Verlag, New York*, 1997.

William C. Brown, A Second Course in Linear Algebra. A Wiley-Interscience Publication. *John Wiley & Sons, Inc., New York*, 1988.

Jonathan S. Golan, The Linear Algebra a Beginning Graduate Student Ought to Know. Second Edition. *Springer, Dordrecht*, 2007.

1. Vector Spaces and Linear Transformations

1.1. **Vector Spaces.** Definitions of vector space, subspace, quotient space, linear independence, direct sum, basis, coordinates, and dimension.

1.2. **Linear Transformations.** Definitions of linear transformation, and its image and kernel. Correspondence between linear transformations and matrices, null space and column space. Gaussian elimination, reduced row echelon form, and rank. The Rank-Nullity Theorem. Solving linear systems.

1.3. **Dual Spaces.** Definition of the dual space V^* of a vector space V, and the dual basis. If V is finite dimensional, V is non-canonically isomorphic to V^*, and V is canonically isomorphic to V^{**}.

2. Eigenvalues and Jordan form

2.1. **Determinant and Eigenvalues.** Definition and computation of determinant, eigenvalues, and eigenvectors. Change of basis and diagonalization. Characteristic and minimal polynomials.

2.2. **Jordan Form.** Definition of semisimple and nilpotent. Jordan form of a matrix and endomorphism.

3. Bilinear Forms

3.1. **Inner Product Spaces.** Inner products, orthogonal and orthonormal vectors, Gram-Schmidt process to find an orthonormal basis.

3.2. **Bilinear Forms.** Definition of bilinear form and non-degeneracy. Symmetric, alternating, and Hermitian forms. Isometries of forms, normal operators.

3.3. **Classification.** Classification of forms over $\mathbb{R}$ and $\mathbb{C}$. Every bilinear form can be uniquely written as the sum of a symmetric and alternating form. Change of basis for a bilinear form, and diagonalization of forms over $\mathbb{R}$ and $\mathbb{C}$.