Integration Workshop 2009
Project descriptions

1. **The Berlekamp algorithm**: For polynomials with coefficients in a finite field the Berlekamp algorithm factors polynomials of high degree in reasonable amounts of time.
2. **The Hopf fibration**: an introduction from the points of view of the quaternions and of the complex numbers to an important object in fields such as topology and Lie groups with many physical applications such as rigid body mechanics and magnetic monopoles.
3. **Conformal maps and SLE**: The relationship betwee the Loewner differential equation and conformal maps with a glimpse of Schramm Loewner evolution (SLE).
4. **Quadratic Reciprocity**: Eisenstein's slick proof of Gauss's Theorema Aureum of quadratic reciprocity.
5. **Multilinear Algebra**: How to show that the div, grad, and curl operators from vector calculus can all be thought of as special cases of a single operator on differential forms.
6. **Tychonoff**: An arbitrary product of compact spaces is compact.
7. **Periodic continued fractions**: A real number has a periodic continued fraction expansion if and only if it satisfies a quadratic equation.
8. **Generating functions**: How to use generating functions to solve linear recurrences.
9. **Nice groups actions on the upper half plane**: The topology and geometry of actions of suitable groups of matrices on the upper half plane.
10. **The space of circles**: Some geometry of the set of all circles in the plane.
11. **The space of metric spaces**: The set of all compact metric spaces can itself be made into a metric space.
12. **Constructing Brownian motion**: Basics on the Wiener process. (Requires somewhat sophisticated analysis.)
13. **Solving elliptic PDEs**: Hilbert space ideas can be used to solve elliptic PDEs.
14. **Lie brackets and parking a car**: The dynamical system associated to a vector field, with applications to control theory.
15. **Quadratic reciprocity**: Eisenstein's proof of the famous result about when a prime is congruent to the square of a number modulo another prime.
16. **Stone-Weierstrass**: Generalizes the fact that a continuous function on a a finite closed interval can be uniformly approximated by polynomials.
17. **Gelfand-Mazur**: Every element of a complex Banach algebra has a non-empty, compact spectrum.
18. **Topological groups and duality**: Basics plus the result that Pontryagin duality interchanges compact and discrete topological groups.
19. **Urysohn metrization theorem**: Characterizes those topological spaces which admit a metric inducing the given topology.
20. **Linear differential equations and the Jordan form**: Knowledge of the Jordan form of a linear transformation leads quickly to a characterization of the solutions of a linear, constant-coefficient differential equation.
21. **Multilinear algebra**: Basics on some of the algebra relevant to the Topology-Geometry core course.
22. **Combinatorial topology of dynamical systems**: How the topology of a surface affects the fixed points of the flow of a vector field.
23. **Geometric aspects of elliptic curves**: Some basics on the complex plane modulo a lattice as a space and as a group.
24. **Group representations**: Basics on groups acting on vector spaces.
25. **Topological vector spaces**: A finte dimensional real vector space has a unique reasonable topology.
26. **The Weierstrass $\wp$-function**: Basics on doubly periodic functions on the complex plane.
27. **Harmonic analysis and the distribution of numbers**: Ideas of Fourier analysis leading to Weyl's theorem that the multiples of an irrational angle are equidistributed in the circle.
28. **Constructing the p-adic numbers**: The p-adics are a completion of the rationals in the same sense that the reals are a completion of the rationals. The project gives two constructions and shows that they yield the same object (a topological field).

29. **Modules over PIDs**: A proof of the fundamental structure theorem for said modules.
30. **Winding numbers**: How many times does a closed curve in the plane wind around a point? Making this precise has amazing applications like the Brouwer fixed point theorem and the fundamental theorem of algebra.
31. **Primes in an arithmetic progression**: Uses analytic functions to show that if a is relatively prime to m, then the sequence $a, a+m, a+2m, \ldots$ contains infinitely many prime numbers.
32. **Compact Hausdorff spaces and C^* algebras**: A compact space determines and is determined by its algebra of functions.
33. **Calculating rational canonical forms**: How to find a nice standard form (different than the Jordan form) of a matrix.