Geometric Decomposition of Serendipity Finite Element Spaces

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Finite Element Exterior Calculus and Applications

What is a serendipity finite element method?

Goal: Efficient, accurate approximation of the solution to a PDE over $\Omega \subset \mathbb{R}^n$.

Standard $O(h^r)$ **tensor product** finite element method in $\mathbb{R}^n$:

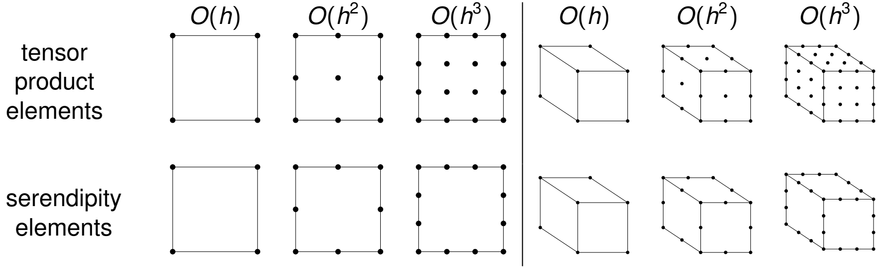
$\to$ Mesh Ω by n-dimensional cubes of side length h.

$\to$ Set up a linear system involving $(r+1)^n$ degrees of freedom (DoFs) per cube.

$\to$ For unknown continuous solution u and computed discrete approximation u_h:

$$\underbrace{||u - u_h||_{H^1(\Omega)}}_{\text{approximation error}} \leq \underbrace{C\,h^r\,|u|_{H^{r+1}(\Omega)}}_{\text{optimal error bound}}, \qquad \forall u \in H^{r+1}(\Omega).$$

A $O(h^r)$ **serendipity** FEM converges at the **same rate** with **fewer DoFs** per element:

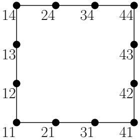


Example: For $O(h^3)$, $d = 3$, 50% fewer DoFs $\to$ $\boxed{\approx 50\% \text{ smaller linear system}}$

What is a geometric decomposition?

A **geometric decomposition** for a finite element space is an explicit correspondence:

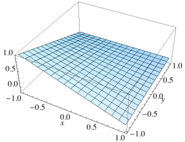
$$\left\{1, x, y, x^2, y^2, xy, x^3, y^3, x^2y, xy^2, x^3y, xy^3\right\} \longleftrightarrow \left\{\vartheta_{11}, \vartheta_{14}, \vartheta_{41}, \vartheta_{44}, \vartheta_{12}, \vartheta_{13}, \vartheta_{42}, \vartheta_{43}, \vartheta_{21}, \vartheta_{31}, \vartheta_{24}, \vartheta_{34}\right\} \longleftrightarrow$$



monomials $\longleftrightarrow$ **basis functions** $\longleftrightarrow$ **domain points**

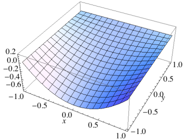
$\rightarrow$ Previously known basis functions employ Legendre polynomials

$\rightarrow$ These functions bear no symmetrical correspondence to the domain points



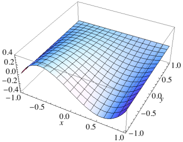
$\frac{1}{4}(x-1)(y-1)$

vertex



$-\frac{1}{4}\sqrt{\frac{3}{2}}\left(x^2-1\right)(y-1)$

edge (quadratic)



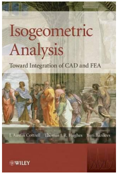
$-\frac{1}{4}\sqrt{\frac{5}{2}}x\left(x^2-1\right)(y-1)$

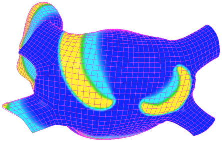
edge (cubic)

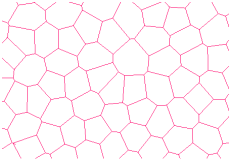
SZABÓ AND I. BABUŠKA *Finite element analysis*, Wiley Interscience, 1991.

Motivations and Related Topics

Goal: Construct geometric decompositions of serendipity spaces using linear combinations of standard tensor product functions. **Focus:** Cubic Hermites.







- **Modern mathematics:** Finite Element Exterior Calculus, Discrete Exterior Calculus, Virtual Element Methods. . .

  ARNOLD, AWANOU *The serendipity family of finite elements*, Found. Comp. Math, 2011.

- **Isogeometric analysis:** Finding basis functions suitable for both domain description and PDE approximation avoids the expensive computational bottleneck of re-meshing.

  COTTRELL, HUGHES, BAZILEVS *Isogeometric Analysis: Toward Integration of CAD and FEA*, Wiley, 2009.

- **Computational Efficiency:** Modern applications such as patient-specific cardiac electrophysiology need efficient, stable, error-bounded 'real-time' methods

- **Flexible Domain Meshing:** Serendipity type elements for Voronoi meshes provides various computational benefits.

  RAND, GILLETTE, BAJAJ *Quadratic Serendipity Finite Elements on Polygons Using Generalized Barycentric Coordinates* arXiv:1109.3259, 2011.

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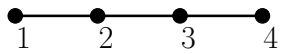
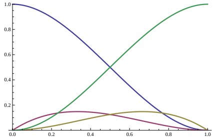
Outline

monomials		basis functions		domain points
$\{1, x, x^2, x^3\}$	$\longleftrightarrow$ *approxim.* $\longleftrightarrow$	$\{\psi_1, \psi_2, \psi_3, \psi_4\}$	$\longleftrightarrow$ *geometry* $\longleftrightarrow$	

Cubic Hermite Basis on $[0, 1]$

$$\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{bmatrix} := \begin{bmatrix} 1 - 3x^2 + 2x^3 \\ x - 2x^2 + x^3 \\ x^2 - x^3 \\ 3x^2 - 2x^3 \end{bmatrix}$$

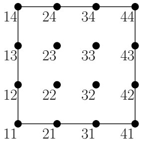


Approximation: $x^r = \sum_{i=1}^{4} \varepsilon_{r,i}\psi_i$, for $r = 0, 1, 2, 3$, where $[\varepsilon_{r,i}] = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & -3 & 1 \end{pmatrix}$

Geometry: $u = u(0)\psi_1 + u'(0)\psi_2 - u'(1)\psi_3 + u(1)\psi_4, \qquad \forall u \in \underbrace{\mathcal{P}_3([0,1])}_{\text{cubic polynomials}}$

Cubic Hermite Geometric Decomposition: 2D

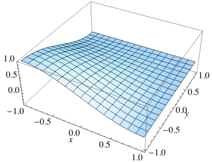
$$\underbrace{\left\{ \begin{array}{c} x^r y^s \\ 0 \le r,s \le 3 \end{array} \right\}}_{\mathcal{Q}_3([0,1]^2)} \longleftrightarrow \left\{ \begin{array}{c} \psi_i(x)\psi_j(y) \\ 1 \le i,j \le 4 \end{array} \right\} \longleftrightarrow$$



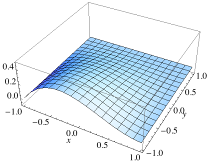
monomials $\longleftrightarrow$ **basis functions** $\longleftrightarrow$ **domain points**

Approximation: $x^r y^s = \displaystyle\sum_{i=1}^{4}\sum_{j=1}^{4} \varepsilon_{r,i}\varepsilon_{s,j}\psi_{ij}$, for $0 \le r,s \le 3$, $\varepsilon_{r,i}$ as in 1D.

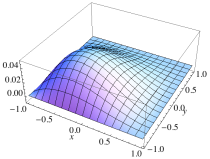
Geometry:



ψ_{11}



ψ_{21}

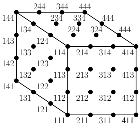


ψ_{22}

$$u = u|_{(0,0)}\psi_{11} + \partial_x u|_{(0,0)}\psi_{21} + \partial_y u|_{(0,0)}\psi_{12} + \partial_x\partial_y u|_{(0,0)}\psi_{22} + \cdots, \qquad \forall u \in \mathcal{Q}_3([0,1]^2)$$

Cubic Hermite Geometric Decomposition: 3D

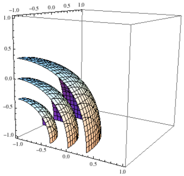
$$\underbrace{\left\{ \begin{array}{c} x^r y^s z^t \\ 0 \leq r, s, t \leq 3 \end{array} \right\}}_{\mathcal{Q}_3([0,1]^3)} \quad \longleftrightarrow \quad \begin{array}{c} \psi_i(x)\psi_j(y)\psi_k(z) \\ 1 \leq i, j, k \leq 4 \end{array} \quad \longleftrightarrow$$



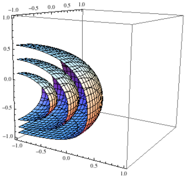
monomials $\longleftrightarrow$ **basis functions** $\longleftrightarrow$ **domain points**

Approximation: $x^r y^s z^t = \sum_{i=1}^{4} \sum_{j=1}^{4} \sum_{k=1}^{4} \varepsilon_{r,i}\varepsilon_{s,j}\varepsilon_{t,k}\psi_{ijk}$, for $0 \leq r, s, t \leq 3$, $\varepsilon_{r,i}$ as in 1D.

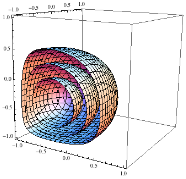
Geometry: Contours of level sets of the basis functions:



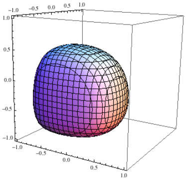
ψ_{111}



ψ_{112}



ψ_{212}



ψ_{222}

Two families of finite elements on cubical meshes

$\mathcal{Q}_r\Lambda^k([0,1]^n) \longrightarrow$ standard tensor product spaces ($\leq$ degree r in each variable)

early work: RAVIART, THOMAS 1976, NEDELEC 1980
more recently: ARNOLD, BOFFI, BONIZZONI arXiv:1212.6559, 2012

$\mathcal{S}_r\Lambda^k([0,1]^n) \longrightarrow$ serendipity finite element spaces (superlinear degree r)

early work: STRANG, FIX *An analysis of the finite element method* 1973
more recently: ARNOLD, AWANOU FoCM 11:3, 2011, and arXiv:1204.2595, 2012.

The **superlinear** degree of a polynomial ignores linearly-appearing variables.

$$n=2: \quad \{\overbrace{\underbrace{1, x, y, x^2, y^2, xy, x^3, y^3, x^2y, xy^2, x^3y, xy^3}_{\mathcal{S}_3\Lambda^0([0,1]^2)\ \text{(dim=12)}}, x^2y^2, x^3y^2, x^2y^3, x^3y^3}^{\mathcal{Q}_3\Lambda^0([0,1]^2)\ \text{(dim=16)}}\}$$

$$n=3: \quad \{\overbrace{\underbrace{1, \ldots, xyz, \ \ x^3y, x^3z, y^3z, \ldots, x^3yz, xy^3z, xyz^3}_{\mathcal{S}_3\Lambda^0([0,1]^3)\ \text{(dim=32)}}, x^3y^2, \ldots, x^3y^3z^3}^{\mathcal{Q}_3\Lambda^0([0,1]^3)\ \text{(dim=64)}}\}$$

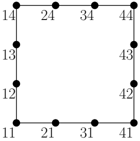
$\mathcal{Q}_r\Lambda^k$ and $\mathcal{S}_r\Lambda^k$ and have the **same** key mathematical properties needed for FEEC
(degree, inclusion, trace, subcomplex, unisolvence, commuting projections)
but for fixed $k \geq 0$, $r, n \geq 2$ the serendipity spaces have **fewer** degrees of freedom

Outline

Cubic Hermite Serendipity Geom. Decomp: 2D

Theorem [G, 2012]: A Hermite-like geometric decomposition of $\mathcal{S}_3([0,1]^2)$ exists.

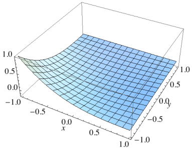
$$\underbrace{\left\{ \begin{array}{c} x^r y^s \\ \text{sldeg} \leq 3 \end{array} \right\}}_{\mathcal{S}_3([0,1]^2)} \longleftrightarrow \left\{ \begin{array}{c} \vartheta_{\ell m} \\ \text{(to be defined)} \end{array} \right\} \longleftrightarrow$$



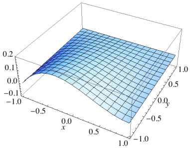
monomials $\longleftrightarrow$ **basis functions** $\longleftrightarrow$ **domain points**

Approximation: $x^r y^s = \sum_{\ell m} \varepsilon_{r,i} \varepsilon_{s,j} \vartheta_{\ell m}$, for superlinear degree$(x^r y^s) \leq 3$

Geometry:



ϑ_{11}



ϑ_{21}

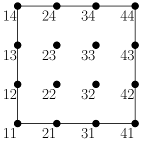
$$\begin{aligned} u &= u|_{(0,0)}\vartheta_{11} \\ &\quad + \partial_x u|_{(0,0)}\vartheta_{21} \\ &\quad + \partial_y u|_{(0,0)}\vartheta_{12} \\ &\quad + \cdots \end{aligned}$$

$\forall u \in \mathcal{S}_3([0,1]^2)$

Proof Overview:

1 Fix index sets and basis orderings based on domain points:

$$\begin{aligned} V &= \text{vertices } (11, 14, \ldots) \\ E &= \text{edges } (12, 13, \ldots) \\ D &= \text{interior } (22, 23, \ldots) \end{aligned} \qquad \longleftrightarrow$$



$$[\vartheta_{\ell m}] := [\ \vartheta_{11}, \vartheta_{14}, \vartheta_{41}, \vartheta_{44},\ \ \vartheta_{12}, \vartheta_{13}, \vartheta_{42}, \vartheta_{43}, \vartheta_{21}, \vartheta_{31}, \vartheta_{24}, \vartheta_{34}\ ],$$

$$[\psi_{ij}] := [\ \underbrace{\psi_{11}, \psi_{14}, \psi_{41}, \psi_{44}}_{\text{indices in } V},\ \ \underbrace{\psi_{12}, \psi_{13}, \psi_{42}, \psi_{43}, \psi_{21}, \psi_{31}, \psi_{24}, \psi_{34}}_{\text{indices in } E},\ \ \underbrace{\psi_{22}, \psi_{23}, \psi_{32}, \psi_{33}}_{\text{indices in } D}\ ]$$

2 Define a 12×16 matrix $\mathbb{H}$ with entries $h_{ij}^{\ell m}$ so that $\ell m \in V \cup E$, $ij \in V \cup E \cup D$.

3 Define the serendipity basis functions $\vartheta_{\ell m}$ via

$$\boxed{[\vartheta_{\ell m}] := \mathbb{H}[\psi_{ij}]}$$

and show that the approximation and geometry properties hold.

Cubic Hermite Serendipity Geom. Decomp: 2D

Proof Details:

$\boxed{\textbf{2}}$ Define a 12×16 matrix $\mathbb{H}$ with entries $h_{ij}^{\ell m}$ so that $\ell m \in V \cup E$, $ij \in V \cup E \cup D$.

$$\mathbb{H} := \left[\begin{array}{c|cccc} & -1 & 1 & 1 & -1 \\ & 1 & -1 & -1 & 1 \\ & 1 & -1 & -1 & 1 \\ & -1 & 1 & 1 & -1 \\ & -1 & 0 & 1 & 0 \\ \mathbb{I} & 0 & -1 & 0 & 1 \\ \text{(12x12 identity matrix)} & 1 & 0 & -1 & 0 \\ & 0 & 1 & 0 & -1 \\ & -1 & 1 & 0 & 0 \\ & 0 & 0 & -1 & 1 \\ & 1 & -1 & 0 & 0 \\ & 0 & 0 & 1 & -1 \end{array}\right]$$

$\boxed{\textbf{3}}$ Define $[\vartheta_{\ell m}] := \mathbb{H}[\psi_{ij}]$. The geometry property holds since for $\ell m \in V \cup E$,

$$\vartheta_{\ell m} = \underbrace{\psi_{\ell m}}_{\text{bicubic Hermite}} + \underbrace{\sum_{ij \in D} h_{ij}^{\ell m} \psi_{ij}}_{\text{zero on boundary}} \implies \vartheta_{\ell m} \equiv \psi_{\ell m} \text{ on edges}$$

Cubic Hermite Serendipity Geom. Decomp: 2D

Proof Details:

To prove that the approximation property holds, observe:

$$[\vartheta_{\ell m}] := \mathbb{H}[\psi_{ij}] \quad \text{implies} \quad \sum_{ij} h_{ij}^{\ell m}\psi_{ij} = \vartheta_{\ell m}$$

For all (r,s) pairs such that $\text{sldeg}(x^r y^s) \leq 3$, the matrix entries in column ij satisfy

$$\varepsilon_{r,i}\varepsilon_{s,j} = \sum_{\ell m \in V \cup E} \varepsilon_{r,\ell}\varepsilon_{s,m} h_{ij}^{\ell m}$$

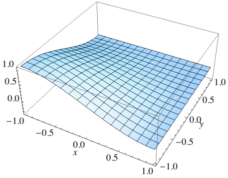
Substitute these into the Hermite 2D approximation property:

$$\begin{aligned} x^r y^s &= \sum_{ij \in V \cup E \cup D} \varepsilon_{r,i}\varepsilon_{s,j}\psi_{ij} = \sum_{ij}\sum_{\ell m} \varepsilon_{r,\ell}\varepsilon_{s,m} h_{ij}^{\ell m}\psi_{ij} \\ &= \sum_{\ell m} \varepsilon_{r,\ell}\varepsilon_{s,m} \sum_{ij} h_{ij}^{\ell m}\psi_{ij} = \sum_{\ell m} \varepsilon_{r,\ell}\varepsilon_{s,m}\vartheta_{\ell m} \end{aligned}$$

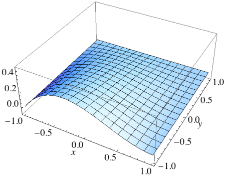
Hence $[\vartheta_{\ell m}]$ is a basis for $\mathcal{S}_2([0,1]^2)$, completing the geometric decomposition. $\square$

Hermite Style Serendipity Functions (2D)

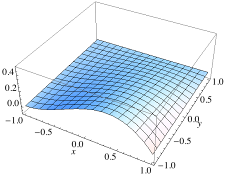
$$[\vartheta_{\ell m}] = \begin{bmatrix} \vartheta_{11} \\ \vartheta_{14} \\ \vartheta_{41} \\ \vartheta_{44} \\ \vartheta_{12} \\ \vartheta_{13} \\ \vartheta_{42} \\ \vartheta_{43} \\ \vartheta_{21} \\ \vartheta_{31} \\ \vartheta_{24} \\ \vartheta_{34} \end{bmatrix} = \begin{bmatrix} -(x-1)(y-1)(-2+x+x^2+y+y^2) \\ (x-1)(y+1)(-2+x+x^2-y+y^2) \\ (x+1)(y-1)(-2-x+x^2+y+y^2) \\ -(x+1)(y+1)(-2-x+x^2-y+y^2) \\ -(x-1)(y-1)^2(y+1) \\ (x-1)(y-1)(y+1)^2 \\ (x+1)(y-1)^2(y+1) \\ -(x+1)(y-1)(y+1)^2 \\ -(x-1)^2(x+1)(y-1) \\ (x-1)(x+1)^2(y-1) \\ (x-1)^2(x+1)(y+1) \\ -(x-1)(x+1)^2(y+1) \end{bmatrix} \cdot \frac{1}{8}$$



ϑ_{11}



ϑ_{21}

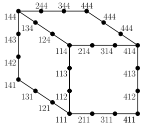


ϑ_{31}

Cubic Hermite Serendipity Geom. Decomp: 3D

Theorem [G, 2012]: A Hermite-like geometric decomposition of $\mathcal{S}_3([0,1]^3)$ exists.

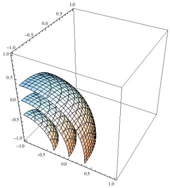
$$\underbrace{\left\{ \begin{array}{c} x^r y^s z^t \\ \text{sldeg} \leq 3 \end{array} \right\}}_{\mathcal{S}_3([0,1]^3)} \quad \longleftrightarrow \quad \left\{ \begin{array}{c} \vartheta_{\ell mn} \\ \text{(to be defined)} \end{array} \right\} \quad \longleftrightarrow$$



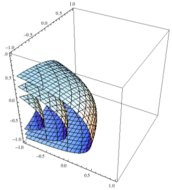
monomials $\longleftrightarrow$ **basis functions** $\longleftrightarrow$ **domain points**

Approximation: $x^r y^s z^t = \sum_{\ell mn} \varepsilon_{r,i}\varepsilon_{s,j}\varepsilon_{t,k}\vartheta_{\ell mn}$, for superlinear degree$(x^r y^s z^t) \leq 3$

Geometry:



ϑ_{111}



ϑ_{112}

$$\begin{aligned} u &= u|_{(0,0,0)}\vartheta_{111} \\ &+ \partial_x u|_{(0,0,0)}\vartheta_{211} \\ &+ \partial_y u|_{(0,0,0)}\vartheta_{121} \\ &+ \partial_z u|_{(0,0,0)}\vartheta_{112} \\ &+ \cdots \end{aligned}$$

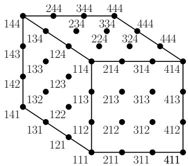
$\forall u \in \mathcal{S}_3([0,1]^3)$

Cubic Hermite Serendipity Geom. Decomp: 3D

Proof Overview:

1 Fix index sets and basis orderings based on domain points:

$$\begin{aligned} V &= \text{vertices } (111, \ldots) \\ E &= \text{edges } (112, \ldots) \\ F &= \text{face interior } (122, \ldots) \\ M &= \text{volume interior } (222, \ldots) \end{aligned} \quad \longleftrightarrow$$



$$[\vartheta_{\ell mn}] := [\ \vartheta_{111}, \ldots, \vartheta_{444}, \ \ \vartheta_{112}, \ldots, \vartheta_{443}\ ],$$

$$[\psi_{ijk}] := [\ \underbrace{\psi_{111}, \ldots, \psi_{444},}_{\text{indices in } V} \ \underbrace{\psi_{112}, \ldots, \psi_{443},}_{\text{indices in } E} \ \underbrace{\psi_{122}, \ldots, \psi_{433},}_{\text{indices in } F} \ \underbrace{\psi_{222}, \ldots, \psi_{333}}_{\text{indices in } M}\ ]$$

2 Define a 32×64 matrix $\mathbb{W}$ with entries $h_{ijk}^{\ell mn}$ (where $\ell mn \in V \cup E$)

3 Define the serendipity basis functions $\vartheta_{\ell mn}$ via

$$\boxed{[\vartheta_{\ell mn}] := \mathbb{W}[\psi_{ijk}]}$$

and show that the approximation and geometry properties hold.

Proof Details:

$\boxed{\mathbf{2}}$ Define a 32×64 matrix $\mathbb{W}$ with entries $h_{ijk}^{\ell mn}$ so that $\ell mn \in V \cup E$

$$\mathbb{W} := \left[\begin{array}{c|c} \mathbb{I} & \text{specific full rank} \\ \text{(32}x\text{32 identity matrix)} & 32x32 \text{ matrix} \\ & \text{with entries -1, 0, or 1} \end{array}\right]$$

$\boxed{\mathbf{3}}$ Define $[\vartheta_{\ell mn}] := \mathbb{W}[\psi_{ijk}]$.

$\longrightarrow$ Confirm directly that $[\vartheta_{\ell mn}]$ restricts to $[\vartheta_{\ell m}]$ on faces.

$\longrightarrow$ Similar proof technique confirms geometry and approximation properties.

$$[\vartheta_{\ell mn}] = \begin{bmatrix} \vartheta_{111} \\ \vartheta_{114} \\ \vdots \\ \vartheta_{442} \\ \vartheta_{443} \end{bmatrix} = \begin{bmatrix} (x-1)(y-1)(z-1)(-2+x+x^2+y+y^2+z+z^2) \\ -(x-1)(y-1)(z+1)(-2+x+x^2+y+y^2-z+z^2) \\ \vdots \\ (x+1)(y+1)(z-1)^2(z+1) \\ -(x+1)(y+1)(z-1)(z+1)^2 \end{bmatrix} \cdot \frac{1}{16}$$

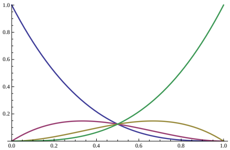
Complete list and more details in my paper:

GILLETTE *Hermite and Bernstein Style Basis Functions for Cubic Serendipity Spaces on Squares and Cubes*, arXiv:1208.5973, 2012

Cubic Bernstein Serendipity Geom. Decomp: 2D, 3D

Cubic Bernstein Basis on $[0,1]$

$$\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{bmatrix} := \begin{bmatrix}(1-x)^3\\ (1-x)^2x\\ (1-x)x^2\\ x^3\end{bmatrix}$$

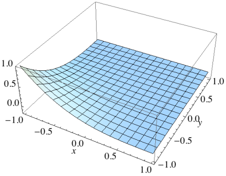


Theorem [G, 2012]: Bernstein-like geometric decompositions of $\mathcal{S}_3([0,1]^2)$ and $\mathcal{S}_3([0,1]^3)$ exist.

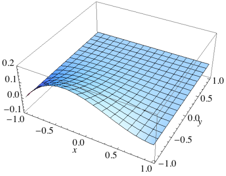
$$[\xi_{\ell m}] = \begin{bmatrix}\xi_{11}\\ \xi_{14}\\ \vdots\\ \xi_{24}\\ \xi_{34}\end{bmatrix} = \begin{bmatrix}(x-1)(y-1)(-2-2x+x^2-2y+y^2)\\ -(x-1)(y+1)(-2-2x+x^2+2y+y^2)\\ \vdots\\ (x-1)^2(x+1)(y+1)\\ -(x-1)(x+1)^2(y+1)\end{bmatrix}\cdot\frac{1}{16}$$

$$[\xi_{\ell mn}] = \begin{bmatrix}\xi_{111}\\ \xi_{114}\\ \vdots\\ \xi_{442}\\ \xi_{443}\end{bmatrix} = \begin{bmatrix}-(x-1)(y-1)(z-1)(-5-2x+x^2-2y+y^2-2z+z^2)\\ (x-1)(y-1)(z+1)(-5-2x+x^2-2y+y^2+2z+z^2)\\ \vdots\\ (x+1)(y+1)(z-1)^2(z+1)\\ -(x+1)(y+1)(z-1)(z+1)^2\end{bmatrix}\cdot\frac{1}{32}$$

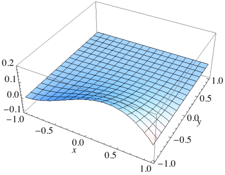
Bernstein Style Serendipity Functions (2D)



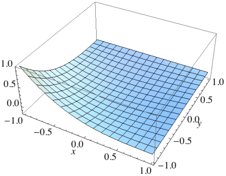
β_{11}



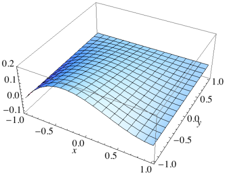
β_{21}



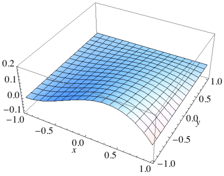
β_{31}



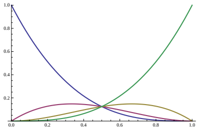
ξ_{11}



ξ_{21}



ξ_{31}



Bicubic Bernstein functions (top) and Bernstein-style serendipity functions (bottom).

$\rightarrow$ Note boundary agreement with Bernstein functions.

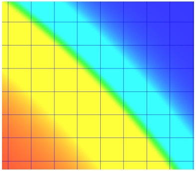
Outline

Application: Cardiac Electrophysiology

→ Initial implementation of cubic Hermite serendipity functions as part of the Continuity software package for cardiac electrophysiology models.

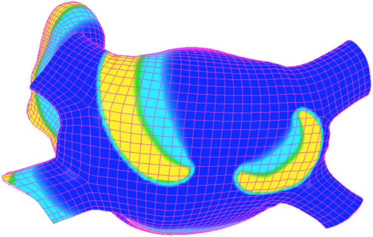
→ Used to solve the monodomain equations, similar to the heat equation:

$$\partial_t u - \Delta u = f$$



→ Cubic Hermite serendipity functions resolve the voltage wavefront equivalently (in eyeball norm) to regular Hermite bicubics.

→ Initial results show a **4x computational speedup** in 3D.



→ Current research into isogeometric analysis with applications to clinical modeling.

Future Directions and Open Questions

$\rightarrow$ Construction of Bernstein-like bases for $\mathcal{S}_r\Lambda^k([0,1]^n)$ for
- $\rightarrow$ Higher order scalar cases ($k = 0$, $r > 3$, $n = 2, 3$)
- $\rightarrow$ Higher form order cases ($k > 0$)

$\rightarrow$ Generalized Bernstein-like construction process:

$$[\vartheta_\ell] := \mathbb{M}\ [\beta_i]$$

$$[\text{ basis for } \mathcal{S}_r\Lambda^k([0,1]^n)\ ] := \mathbb{M}\ [\text{ Bernstein basis for } \mathcal{Q}_r\Lambda^k([0,1]^n)\ ]$$

Define entries of $\mathbb{M}$ so that the appropriate approximation and geometry properties hold for $[\vartheta_s]$.

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Slides and pre-prints: `http://ccom.ucsd.edu/~agillette`