Serendipity Methods: Using Mathematics to Accelerate Computation

BY ANDREW GILLETTE

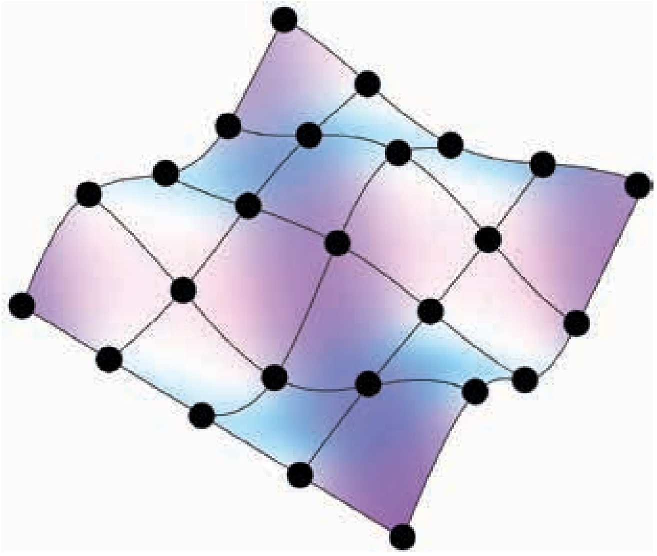
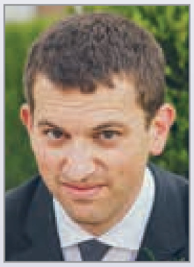


Figure A. Building a surface model by a standard approach: look up the terrain height at all 25 grid points.



When you hear the term "serendipity" in reference to scientific research, any number of essential discoveries may come to mind. When the term "serendipity" is used in the mathematical community of numerical analysts, however, it refers to a particular computational methodology discovered in the 1970s whose unexpected effectiveness is indeed serendipitous. The term "serendipity finite element method" has appeared in engineering textbooks and software documentation for the past 40 years, and scientists have leveraged the method for at least as long. Remarkably, a mathematical understanding of this phenomenon is still being developed today.

To get an idea of the serendipity finite element method, imagine that you want to build a high-resolution model of the surface of the Tucson Catalina Mountains, based on a flat map of the area. Suppose your map is partitioned into a grid of 1 mile by 1 mile blocks. You decide to divide each of these square blocks into 16 sub-blocks that are 1/4 mile on each side. You then look up the height at the corner of each small block and build a surface model of the terrain (see Figure A). The serendipity method reveals that you could construct a surface model of the terrain that is "just as good," in a certain mathematically precise sense, by looking up the heights of a much smaller number of points: the ones along the perimeter of each square mile block and just the point at the center of each square mile block (see Figure B). In this way, you can spend much less time building a surface representation without degrading the quality of the result - a truly serendipitous occurrence!

In scientific applications, the serendipity method is most useful when the object you are trying to model is not something so visible as a mountain range. Astronomers use the finite element method to model extremely large-scale events, such as the development of galaxies over billions of years. Biologists use the finite element method to model extremely small-scale events, such as the diffusion of calcium ions during the contraction of a muscle fiber. Engineers use the finite element method for a wide variety of problems, such as evaluating the stress on bridges and roadways. In any of these contexts, serendipity methods reduce the computational effort and resources needed to gain new insights from the models being developed. Mathematicians now face the challenging and exciting task of explaining why the serendipity method is not just a matter of "blind luck" but in fact a rigorous method derived from a thorough understanding of approximation theory and numerical analysis.

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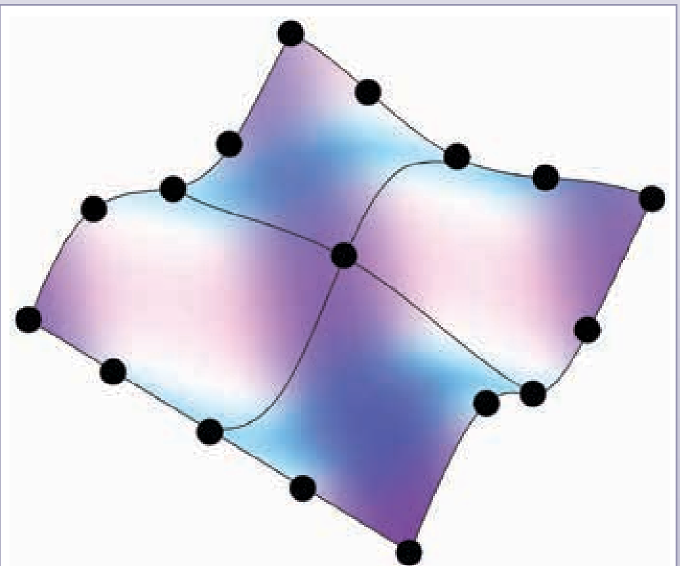

Figure B. Building a surface model via the serendipity finite element method: look up the terrain height only at perimeter grid points and at the central grid point.