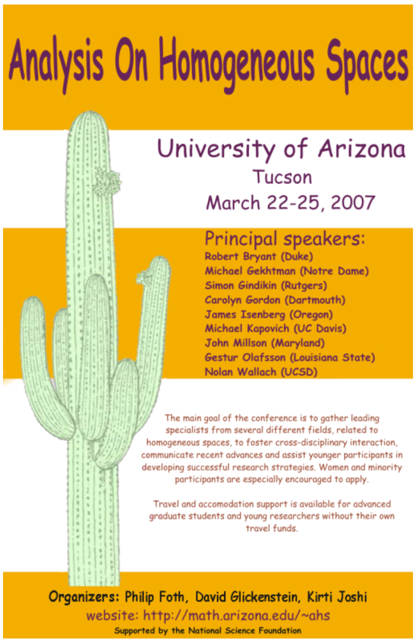
Analysis on Homogeneous Spaces

March 22-25, 2007, Tucson, Arizona

Organizers: Philip Foth, David Glickenstein and Kirti Joshi

Abstracts of Talks



http://www.math.arizona.edu/~ahs

Contents

1 J. Behrstock

Speaker: Jason Behrstock, University of Utah.

Title: Dimension and rank of mapping class groups.

Abstract: We will discuss recent work with Yair Minsky towards understanding the large scale geometry of the mapping class group. In particular, we'll explain how to obtain various topological properties of the asymptotic cone of the mapping class group including a computation of its dimension. An application of this analysis is an affirmative solution to Brock-Farb's Rank Conjecture which asserts that MCG has quasi-flats of dimension N if and only if it has a rank N free abelian subgroup.

2 M. Gekhtman

Speaker: Michael Gekhtman, University of Notre Dame.

Title: Semiconductor nets, cluster algebras and integrable systems

Abstract:

A. Postnikov recently constructed a natural map from a planar directed network in a disk to a totally nonnegative Grassmannian. In this construction a choice of a planar directed graph corresponds to a choice of special basic set of Pl\"ucker coordinates. Elementary transformations of planar directed graphs correspond to special cluster transformations in a natural cluster algebra of Grassmannian.

We introduce a Poisson structure on the space of conductivities (weights) of a planar directed network. It induces the Poisson structure compatible with Grassmannian cluster algebra structure. The construction can be extended to networks on higher genus surfaces, i. p. on an annulus. In this case the Poisson structure above can be naturally iterpreted in terms of trigonometric R-matrices. A relation between this construction and a class of integrable systems will be discussed as weel as the notion of minimality for directed graphs on an annulus and the inverse problem of restoring the network from the complete set of boundary measurements of this network. This is a joint work with M. Shapiro and A. Vainshtein.

3 C. Gordon

Speaker: Carolyn Gordon, Dartmouth

Title: A torus action method for constructing manifolds with the same spectral data

Abstract: We address the question: To what extent does spectral data determine the geometry of a Riemannian manifold? In the compact setting, the spectral data considered will be the eigenvalue spectrum of the Laplacian. In the noncompact setting, we will consider scattering data. We describe a method involving torus action for constructing metrics with the same spectral data but with different local geometry. The technique also gives isospectral/isoscattering potentials for the Schrodinger operator.

4 S. Gindikin

Speaker: Simon Gindikin, Rutgers University.

Title: Harmonic analysis on symmetric spaces from point of view of complex analysis.

Abstract:
The modern harmonic analysis was started by E. Cartan and H. Weyl on two quite different ways: algebraic and transcendental (analytical). In the first one E. Cartan considered Lie algebras, in the second one H. Weyl didn't found direct tools to work with complex Lie groups but using the unitary trick transfered problems to compact groups. Appropriate methods of multidimensional complex analysis didn't exist in that time but today such possibilities exist and we'll discuss what this "third way" gives for old and new problems of harmonic analysis.

5 R. Hladky

Speaker: Robert Hladky, University of Rochester.

Title: Minimal and isoperimetric surfaces in Carnot groups.

Abstract: A Carnot group is a nilpotent, stratified Lie group endowed with a subRiemannian metric induced from the stratified structure of the Lie algebra. These structures arise naturally as tangent spaces to subRiemannian manifolds; the geometric environments for studying subelliptic pde. Associated to this subRiemannian metric is a hypersurface measure. We shall discuss the minimal and isoperimetric surface problems associated to this measure, their geometric properties and characterization as solutions to non-linear subelliptic equations. (Depending on length of the talk, I may also discuss applications to problems in neuroscience and computer imaging.)

6 J. Isenberg

Speaker: Jim Isenberg, University of Oregon.

Title: Ricci Flow of Homogeneous Geometries and Einstein Evolution of Spatially Homogeneous Cosmologies

Abstract: Hamilton's scenario for proving the Thurston Geometrization Conjecture suggests that after suitable surgeries, the Ricci flow on components of any given 3 manifold with any given metric should approach the Ricci flow of locally homogeneous 3 geometries. This has motivated the study of the Ricci flow of such geometries. The expected behavior of Ricci flow singularities also motivates the study of these model flows. We recall how to set up the analysis of the Ricci flow of homogeneous geometries, and discuss the behavior of these flows in 3 and 4 dimensions. We also discuss some aspects of the stability of Ricci flows near homogenous geometries.

The Cosmological Principle, which suggests that the universe is at some scale spatially homogenous and isotropic, motivates the study of solutions of the Einstein gravitational field equations on 4 dimensional space-time manifolds which are spatially homogeneous. This analysis, like that of the Ricci flow on homogeneous geometries, reduces to an ODE system for components of the metric. We show to carry out this reduction for the Einstein equations, and discuss the behavior of the solutions. We point out both the profound differences and the remarkable similarities between the Einstein cosmological solutions and the Ricci flow solutions.

7 M. Kapovich

Speaker: M. Kapovich, UC Davis.

Title: Projections in symmetric spaces and buildings.

Abstract: In my talk I will discuss solution of fixed-point problems for certain self-maps of symmetric spaces and buildings.
These fixed-point problems correspond to the restriction problem in the representation theory.
I will also explain how (in the case of Levi subgroups) ideal polygons in symmetric spaces and buildings relate to these fixed-point problems.

8 S. Koshkin

Speaker: Sergiy Koshkin

Title: Homogeneous spaces and the gauge theory

Abstract: Let G be a compact Lie group and H a closed subgroup. We define an analog of the right-invariant Maurer-Cartan form on the homogeneous space G/H and develop gauge theory on the pullbacks of the quotient bundles $G\to G/H$. It turns out that many constructions on the trivial bundles ($H=\{1\}$) generalize to this case. For example, there is a distinguished reference connection, connections can be represented by Lie algebra valued forms and there are familiar looking expressions for gauge action and curvature in these terms.

Moreover, maps into G/H can be encoded by the gauge equivalence classes of connections on pullback bundles. This provides a natural framework for considering coset models of quantum physics and we apply our gauge calculus to the Faddev-Skyrme models with the target manifold being a symmetric space.

9 W. Meeks

Speaker: William Meeks, University of Massachusetts

Title for Talk: The geometry of complete embedded minimal and constant mean curvature surfaces in a complete homogeneous 3 manifold.

Abstract: I will explain some of my recent joint work with others on the geometry of complete embedded minimal and constant mean curvature surfaces in a complete homogenous 3 manifold. One consequence of this study is that all finite topology examples of non-zero constant mean curvature have bounded Gaussian curvature and this restriction leads to interesting topological obstructions and classification results.

10 T. Melcher

Title: Heisenberg group heat kernel inequalities
Speaker: Tai Melcher, University of Virginia.

Abstract:
We will discuss the existence of ``$L^p$-type" gradient estimates for the heat kernel of the natural hypoelliptic ``Laplacian" on the real three-dimensional Heisenberg Lie group. Stochastic calculus methods show that these estimates hold in the case $p>1$. The gradient estimate for $p=2$ implies a corresponding Poincar\'{e} inequality for the heat kernel. The gradient estimate for $p=1$ is still open; if proved, this estimate would imply a logarithmic Sobolev inequality for the heat kernel.

11 J. Millson

Speaker: John Millson, University of Maryland.

Title: Generalized Kostant convexity theorems, the constant term map on spherical Hecke algebras and branching to Levi subgroups

Abstract:

My lecture will give an overview of joint work with Tom Haines and Misha Kapovich. The lecture of Misha Kapovich at this conference will give details of the most interesting of the results detailed below (for example the saturation theorem).

I will discuss four problems related to a pair (G,L) where L is a (reductive) Levi subgroup of a simple Lie group G. For the last two problems we will assume G is split over the rationals. Let P be a parabolic (defined over the rationals) with Levi equal to L. Let T be a common maximal torus for G and L.

The first two problems are the problems of generalizing Kostant's linear and nonlinear convexity theorems from the case where L is a maximal torus to a general Levi. The third problem is the problem of computing the constant term map from the spherical Hecke algebra of G to that of L. The fourth problem is the problem of computing the restriction map of finite dimensional representations from the Langlands' dual of G to the Langlands' dual of L.

If we restrict to integral orbits for the first two problems then the input data for all four problems is the same (a G- dominant cocharacter a of T, an L-dominant cocharacter b of T). Our first main theorem is that the solution set of Problem 4 is a subset of the solution set of Problem 3 which is in turn a subset of the common solution set of Problems 1 and 2. Our next main theorem says that we can reverse the two inclusions if we are willing to saturate by an explicit constant k(G) = at most 2 for the classical groups - for example if a, b is a solution of the second problem then k(G)a, k(G)b is a solution of the third problem.

These results are the analogues of the results for the triangle inequalities/ tensor product decomposition problems of Kapovich-Leeb-Millson (to appear in Memoirs of the AMS.

The Heat equation on finite and infinite dimensional symmetric spaces

Gestur Ólafsson
Department of Mathematics
Louisiana State University

Let $\Delta = \sum \partial^2/\partial x_i^2$ be the Laplace operator on $\mathbb{R}^n$. The *heat equation* is given by

$$\begin{aligned} \Delta u(x,t) &= \frac{\partial}{\partial t} u(x,t) \\ \lim_{t \to 0^+} u(x,t) &= f(x)\ ,. \end{aligned}$$

The solution $u(x,t) = e^{t\Delta} f(x) = H_t f(x)$ is given by

$$H_t f(x) = \int_{\mathbb{R}^n} f(y) h_t(x-y)\, dy = \frac{1}{(4\pi t)^{n/2}} \int_{\mathbb{R}^n} f(y) e^{-(x-y)\cdot(x-y)/4t}\, dy \tag{1}$$

where $h_t(x) = (4\pi t)^{-n/2} e^{-x\cdot x/4t}$ is the heat kernel, i.e. the solution corresponding to $f = \delta_0$. Denote by $d\mu_t(x) = h_t(x)dx$ the *Heat kernel* measure on $\mathbb{R}^n$. Then $H_t f$ makes sense for $f \in L^2(\mathbb{R}^n, d\mu_t^n)$ and $H_t f$ extends to a holomorphic function on $\mathbb{C}^n$. Let $d\sigma_t(z) = (2\pi t)^n e^{-|z|^2/2t}$ denote the Heat kernel measure on $\mathbb{C}^n$ and $\mathcal{F}_t(\mathbb{C}^n) = \mathcal{O}(\mathbb{C}^n) \cap L^2(\mathbb{C}^n, d\sigma_t)$. It is a classical result due to Segal and Bargmann that $H_t : L^2_t(\mathbb{R}^n) = L^2(\mathbb{R}^n, d\mu_t) \to \mathcal{F}_t(\mathbb{C}^n)$ is an unitary isomorphism. Both μ_t and σ_t are probability measures that behave nicely under the canonical projection $\pi_{n+1,n} : \mathbb{R}^{n+1} \to \mathbb{R}^n$, respectively $\pi^{\mathbb{C}}_{n+1,n} : \mathbb{C}^{n+1} \to \mathbb{C}^n$ giving rise to a inductive system:

$$\begin{array}{ccccccc} \cdots & L^2(\mathbb{R}^{n-1}, d\mu_t^{n-1}) & \xrightarrow{\pi^*_{n,n-1}} & L^2_t(\mathbb{R}^n) & \longrightarrow \overset{\pi^*_{j+1,j}}{\cdots \longrightarrow} & \cdots & L^2(\mathbb{R}^\infty, d\mu_t^\infty) \\ & \Big\downarrow H_t^{n-1} & & \Big\downarrow H_t^n & & & \Big\downarrow H_t^\infty \\ \cdots & \mathcal{F}_t(\mathbb{C}^{n-1}) & \xrightarrow{(\pi^{\mathbb{C}}_{n,n-1})^*} & \mathcal{F}_t(\mathbb{C}^n) & \longrightarrow \overset{(\pi^{\mathbb{C}}_{j+1,j})^*}{\cdots \longrightarrow} & \cdots & \mathcal{F}_t(\mathbb{C}^\infty, d\sigma_t^\infty) \end{array} \tag{2}$$

It is a natural problem to find similar results for symmetric spaces. The case where we have a tower $\ldots \subset G_j \subset G_{j+1} \subset \ldots$, G_j a Levi factor of a parabolic subgroup in G_{j+1}, the upper part of (2) and some results on the heat equation were discussed by A. Sinton in his Thesis.

In this talk we will give a breve overview of the work that has been done on the heat equation on semisimple Riemannian symmetric spaces of the noncompact type and then discuss the genearlization of (2). A tutorial lecture discussing some of this material can by found at my webpage:

`www.math.lsu.edu/~olafsson/pdfhbox to1emfiles/ht.pdf`

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13 T. Payne

THE RICCI FLOW FOR NILMANIFOLDS

TRACY PAYNE
Idaho State University

Abstract.

A *nilmanifold* is a homogeneous Riemannian manifold of the form (N, g), where N is a nilpotent Lie group and g is a left-invariant metric on N. We describe the Ricci flow for simply connected nilmanifolds.

We set up a system of ODE's for the Ricci flow for a nilmanifold (N, g), using a change of variables to write the system in terms of a symmetric matrix U in $\mathfrak{gl}_m(\mathbb{Z})$ naturally associated to N. We describe qualitative features of the Ricci flow, such as the rate of decay of the sectional curvature and the "collapsing" of metrics. Nonabelian nilmanifolds do not admit Einstein metrics; the best one can hope for is a soliton metric. We show that if a left-invariant soliton metric g^* does exist on a nilpotent Lie group N, then any other metric g on N of a certain form converges under the Ricci flow to g^*, modulo rescaling, as time goes to infinity.

We define a simultaneous projectivized Ricci flow ψ_t on the space $\mathcal{N}_n$ of all volume-normalized nilmanifolds of fixed dimension n, and we analyze the topological dynamics of that flow, describing a stratification of $\mathcal{N}_n$ by closed invariant sets and finding quantities that are monotonic under the flow. We show that if a nilpotent Lie group N does not admit a soliton metric, then for any initial left-invariant volume-normalized metric $\tilde{g}$ on N, under the flow ψ_t, the nilmanifold $(N, \tilde{g})$ asymptotically approaches a nilmanifold $(N_\infty, \tilde{g}_\infty)$, where $\tilde{g}_\infty$ is a volume-normalized soliton metric on a nilpotent group N_∞ not isomorphic to N.

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14 E. Proctor

Title: An isospectral deformation on an orbifold quotient of a nilmanifold.

Speaker: Emily Proctor, Middlebury College.

Abstract: We construct a Laplace isospectral deformation of metrics on an orbifold quotient of a nilmanifold. Each orbifold in the deformation contains singular points with order two isotropy. Isospectrality is obtained by modifying a generalization of Sunada's Theorem due to DeTurck and Gordon.

15 B. Rubin

Invariant Functions on Grassmannians and the Busemann-Petty Problem

B. Rubin (LSU)

Let K be the group of orthogonal transformations of the Euclidean space $\mathbb{R}^n = \mathbb{R}^\ell \times \mathbb{R}^{n-\ell}$, which preserve the coordinate subspaces $\mathbb{R}^\ell$ and $\mathbb{R}^{n-\ell}$. We show that every K-invariant function on the Grassmann manifold of i-dimensional subspaces ξ of $\mathbb{R}^n$ is completely determined by canonical angles between ξ and $\mathbb{R}^\ell$. We also consider the lower dimensional Busemann-Petty problem which asks, whether origin-symmetric convex bodies in $\mathbb{R}^n$ with smaller i-dimensional central sections necessarily have smaller volumes. We give complete solution to this problem for bodies with K-symmetry.

16 J. Ryan

Speaker: Jon Ryan, University of Arkansas.

Title: Dirac type operators on spin manifolds associated with generalized arithmetic groups

Abstract: Discrete subgroups of generalized arithmetic groups acting on upper half space in n dimensional upper half euclidean space are used to construct examples of conformally flat spin manifolds. A construction due to ahlfors is used to construct fundamental solutions to Dirac type operators. These in turn are used to develop a Hardy space theory and to look at boundary value problems in this context. Automorphic forms are used in these constructions. We end with a look at the same constructions with respect to the hyperbolic metric.

17 C. Seaton

Speaker: Christopher Seaton. Rhodes College.

Title: Generalized orbifold characteristic classes for orbifolds

Abstract:

In many cases, the Chern-Weil descriptions of characteristic classes for vector bundles over smooth manifolds generalize to the case of orbifold vector bundles over smooth orbifolds. However, for bad orbifold vector bundles, vector bundles where a finite group acts trivially on the base but not on the fiber, these definitions are unavailable. We discuss a method of generalizing the descriptions of characteristic classes as well as those of orbifold characteristic classes to these cases.

ON SOME SERIES AND INTEGRALS RELATED TO GROUPS $SO(2,1)$ AND $SO(2,2)$

A. I. NIZHNIKOV AND A. I. SHILIN

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Let σ be a complex number, p and q are real numbers, $p \geq q$, X be a subset of elements of $\mathbb{R}^{p+q}$ satisfying the condition $\sum_{i=1}^{p} x_i^2 - \sum_{i=1}^{q} x_{p+i}^2 = 0$ and D_σ be a linear space consisting of functions $f : D_\sigma \longrightarrow \mathbb{C}$ such that, at first, f be a continuous function on its domain and, at second, $f(\alpha x) = \alpha^\sigma f(x)$. Let $GL(D_\sigma)$, as usually, be a multiplicative group of automorphisms of D_σ. For every $g \in SO(p,q)$ we have the automorphism $T_\sigma : f(x) \longmapsto f(g^{-1}x)$ of D_σ. Thus, $g \longmapsto T_\sigma$ is the representation of $SO(p,q)$.

The matrix elements of above representation and its subrepresentations is considered. Some formulas for series and integrals containing some special functions are obtained.

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19 K. Tapp

Speaker: Kristopher Tapp, Williams College.

TITLE: Quasi-positive curvature on homogeneous bundles

ABSTRACT: I will describe new examples of manifolds which admit a Riemannian metric with sectional curvature nonnegative, and strictly positive at a point. My examples include the unit tangent bundles of CP^n, HP^n and the Cayley plane, and a family of lens space bundles over CP^n. All examples are homogeneous bundles over homogeneous spaces. I will also address the following related question: given a compact Lie group G, classify the left-invariant metrics on G with nonnegative sectional curvature.

20 N. Wallach

Speaker: N. Wallach

Title: Generalized Whittaker Models for Degenerate Principal Series

Abstract: Let G be a real reductive group and P a parabolic subgroup with nilradical N. Let χ be a generic unitary one dimensional representation of N. If σ is an irreducible finite dimensional representation of P then we form the smooth representation $I_\infty(P,\sigma)$ and consider the space of all continuous N intertwining operators from this representation to the one dimensional representation χ. For a class of P (only depending on the complexification of their Lie algebras) we give a complete description in terms of generalized Jacquet integrals (whose properties are also completely described). The class contains all parabolics for which this problem has here-to-fore been solved. K. Baur and I have given a complete classification of these parabolics.