Dam Problem for Math 129

Calculate the total force on the dam shown below:

Solution: We divide the dam into horizontal strips, so $y$ will be the variable of integration, and we set it up so that $y$ increases with the depth of water, which simplifies the calculations. The width, $x$, is a linear function of $y$ and since $x = 3600$ when $y = 0$, and $x = 3000$ when $y = 100$ we have

$$x = 3600 - 6y.$$  

Using the SI acceleration due to gravity and the mass of water

$$\delta = 1000 \frac{\text{kg}}{\text{m}^2}$$

we get

$$\Delta A = (3600 - 6y) \Delta y$$

$$\Delta F = 1000 \cdot 9.8y(3600 - 6y)$$

$$F = 9800 \int_{0}^{100} y(3600 - 6y)dy$$

$$= 9800 \left(1800y^2 - 2y^3\right|_{0}^{100}$$

$$= 9800(1800 \times 10^4 - 2 \times 10^6)$$

$$= 9.8 \times 10^3(18 \times 10^6 - 2 \times 10^6)$$

$$= 9.8 \times 16 \times 10^9 = 1.56 \times 10^{11} \text{ newtons.}$$