The problem is to find the arc-length of the curve

\[ y = x^{\frac{3}{2}} \]

from \((1, 1)\) to \((2, 2\sqrt{2})\). This is clearly an invertible function over the range specified, so we are able to do the problem two ways.

We first obtain

\[ \frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}} \]

and thus the arc-length from \(x = 1\) to \((2, 2\sqrt{2})\) is

\[ L = \int_{1}^{2} \sqrt{1 + \frac{9}{4}x^2} \, dx \]

Using u-substitution (what?) and changing the limits we get

\[ L = \frac{4}{9} \int_{\frac{2}{3}}^{\frac{22}{3}} u^\frac{3}{2} \, du \]

which becomes

\[ \frac{22\sqrt{22} - 13\sqrt{13}}{27} \]

Since \(x = y^{2/3}\) and

\[ \frac{dx}{dy} = \frac{2}{3} y^{-\frac{1}{3}} \]

we get
\[ L = \int_{1}^{2^{\sqrt{2}}} \sqrt{1 + \frac{4}{9}y^{-\frac{3}{8}}} dy \]
\[ = \frac{1}{3} \int_{1}^{2^{\sqrt{2}}} y^{-\frac{1}{10}} \sqrt{4 + 9y^\frac{3}{8}} dy \]

The \( u \)-substitution

\[ u = 9y^\frac{3}{8} + 4 \]
\[ du = 6y^{-\frac{1}{8}} dy \]

thus \( L = \frac{1}{18} \int_{13}^{22} u^\frac{5}{2} du \]
\[ = \frac{1}{27} u^\frac{5}{2} \bigg|_{13}^{22} \]
\[ = \frac{1}{27} [22^\frac{5}{2} - 13^\frac{5}{2}] \]

which becomes
\[ \frac{22\sqrt{22} - 13\sqrt{13}}{27} \]

as before.

The integration in the second solution is a bit more involved. In problems where there is a choice of the variable of integration, a little algebraic foresight may save you some work.