Demonstration that Selection
of
Success and Failure
is
Arbitrary

A manufacturer of polygraph machines estimates that the probability of the latest model detecting a lie is 0.9000. The machine will be tested on a series of ten questions. If it correctly identifies at least eight responses as truth or falsehood, the Tacna city council will buy the thing, else not.

a. What is the probability that the machine will be bought if it is absolutely worthless, i.e. the probability of a correct response is 0.5000?

b. What is the probability that the machine will be rejected even if if performs as the manufacturer claims.

c. What would be the mean number of successes and its standard deviation if the manufacturer's claim were correct?

Solution One: Let a success be a correct identification.

a. I need the probability of eight, nine, or ten responses from $B(10, 0.5)$. This is

$$P(8) + P(9) + P(10) = 0.043945 + 0.009765 + 0.0009765 = .05468$$

b. I need the complement of the probability of eight, nine, or ten successes from $B(10, 0.9)$, or

$$1-(P(8)+P(9)+P(10)) = 1-0.1937-0.38742+0.348678 = 1-0.9298 = 0.07019$$

or cdf up to 7, also 0.07019.

c. The mean, $\mu = np = 0.9 \cdot 10 = 9$ and $\sigma = \sqrt{npq} = \sqrt{10 \cdot 0.9 \cdot 0.1} = \sqrt{.9} = 0.94868$

Solution Two: Let a success be an incorrect identification.

a. I need the probability of zero, one, or two successes from $B(10, 0.5)$. This is

$$P(0) + P(1) + P(2) = 0.0009765 + 0.009765 + 0.043945 = .05468$$

It is also the cdf up to 2, also 0.05468.

b. I need the complement of the probability of zero, one, or two successes from $B(10, 0.1)$, or

$$1-(P(0)+P(1)+P(2)) = 1-(0.348678+0.38742+0.1937) = 1-0.9298 = 0.07019$$

c. The mean, $\mu = np = 0.1 \cdot 10 = 1$ and $\sigma = \sqrt{npq} = \sqrt{10 \cdot 0.9 \cdot 0.1} = \sqrt{.9} = 0.94868$