Math 112 Final Exam Study Aid

Note.
This is the third problem set of the Math 112 study aid. The formulas at the end of this problem set will be identical to the formulas given on the final exam.

Problem Set #3

1. If \( f(x) = 2^x + 3x \), find \( f(-2) \).

   \[
   \begin{align*}
   (A) & \quad -\frac{23}{4} \\
   (B) & \quad -10 \\
   (C) & \quad -\frac{5}{4} \\
   (D) & \quad -4 \\
   (E) & \text{None of these}
   \end{align*}
   \]

2. Which of the following is the piecewise equation for the graph below?

   \[
   \begin{align*}
   (A) & \quad f(x) = \begin{cases} 
   -x^2 & \text{for } x < 1 \\
   1/2 & \text{for } x \geq 1
   \end{cases} \\
   (B) & \quad f(x) = \begin{cases} 
   1/2 & \text{for } x > 1 \\
   |x| & \text{for } x < 1
   \end{cases} \\
   (C) & \quad f(x) = \begin{cases} 
   -|x| & \text{for } x < 1 \\
   1/2 & \text{for } x \geq 1
   \end{cases} \\
   (D) & \quad f(x) = \begin{cases} 
   1/2 & \text{for } x > 1 \\
   -|x| & \text{for } x < 1
   \end{cases} \\
   (E) & \text{None of these}
   \end{align*}
   \]
3. Which of the following has a domain of all real numbers except 18?

(1) \( f(x) = \sqrt{x - 18} \)  
(2) \( g(x) = \frac{2x}{x - 18} \)  
(3) \( h(x) = \frac{1}{x^2 - 324} \)

(A) 1 only  
(B) 2 only  
(C) 2 and 3 only  
(D) 1 and 2 only  
(E) All of them

4. Which of the following graphs represent \( y \) as a function of \( x \)?

(A) All of them  
(B) None of them  
(C) 2, 3, and 4 only  
(D) 2 and 3 only  
(E) 1 only
5. What is the RANGE of the function graphed below?

(A) \([-4, 3]\]  (B) \([-5, -2]\]  (C) \([-2, 3]\]  (D) \([-5, 1]\]  (E) None of these

6. Determine the intervals over which the function graphed below is increasing and decreasing.

Which of the following statements is TRUE about this function?

(A) The function is increasing on \((-2, 3)\); decreasing on \((-1, 3)\)
(B) The function is increasing on \((-3, -1)\) and \((2, 3)\); decreasing on \((-1, 2)\)
(C) The function is increasing on \((-1, 2)\); decreasing on \((-3, -1)\) and \((2, 3)\)
(D) The function is increasing on \((-1, 1)\); decreasing on \((-1, 2)\)
(E) The function is increasing on \((-3, 0)\) and \((1, 3)\); decreasing on \((0, 1)\)
Search and Rescue teams are used in remote areas in the West to find lost people. Experience has shown the team’s chance of finding an individual is a function of the distance by which team members are separated. The percentage found for various separation distances is shown in the table below.

<table>
<thead>
<tr>
<th>Separation distance (ft)</th>
<th>Percent found</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>90</td>
</tr>
<tr>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>60</td>
<td>70</td>
</tr>
<tr>
<td>80</td>
<td>60</td>
</tr>
<tr>
<td>100</td>
<td>50</td>
</tr>
</tbody>
</table>

Find an equation to express the percent found, \( P \), as a function of the separation distance, \( d \), of the team members.

(A) \( P = 0.5d + 100 \)  
(B) \( P = 290 - 0.5d \)  
(C) \( P = 100 - 0.5d \)  
(D) \( P = 90 - 5d \)  
(E) \( P = 2d + 50 \)
8. The graph of $y = f(x)$ is shown below

This is the graph of $y = f(x)$

The graph below is a transformation of the graph of $y = f(x)$. Which of the following is the formula for the function graphed below?

(A) $y = f(-x) - 2$  
(B) $y = -f(x) - 2$  
(C) $y = -f(x + 2)$  
(D) $y = -f(x) + 2$  
(E) $y = f(-x + 2)$

9. If $(5, -6)$ is a point on the graph of $y = g(x)$, which of the following must be a point on the graph of $y = -g(x) + 1$?

(A) $(-5, -6)$  
(B) $(-5, 7)$  
(C) $(5, 6)$  
(D) $(5, 7)$  
(E) $(-6, 6)$
10. Given \( f(x) = 4x + 1 \) and \( g(x) = 15x + 2l \), find \( \left(\frac{f}{g}\right)(-4) \).

(A) \(-\frac{15}{22}\)  (B) \(\frac{17}{22}\)  (C) \(-\frac{5}{6}\)  (D) \(-4\)  (E) None of these

11. Given \( f(x) = 3 + 2x \) and \( h(x) = \sqrt{x} \), find \( (f \circ h)(x) \).

(A) \(\sqrt{3 + 2x}\)  (B) \(3 + 2\sqrt{x}\)  (C) \(\sqrt{3} + \sqrt{2x}\)

(D) \(3\sqrt{x} + 2x\sqrt{x}\)  (E) None of these

12. Which of the following lines is PARALLEL to \(3x - 4y = 7\)?

(A) \( y = 3x - 7 \)

(B) \( y = -\frac{3}{4}x + 8 \)

(C) \( y = -\frac{4}{3}x - 2 \)

(D) \( y = -\frac{4}{3}x - 3 \)

(E) None of these
13. For the function \( f(x) = -x^2 + 6x + 62 \), which of the following statements is/are true?

   (1) the \( x \)-coordinate of the vertex is \(-3\)
   (2) \( f(x) \) has a maximum value of 71
   (3) one of the \( x \)-intercepts is \( \left(3 + \sqrt{71}, 0\right)\)

   (A) 1 and 2 only  (B) 1 and 3 only  (C) 2 and 3 only  
   (D) 2 only  (E) 1 only

14. Find the vertex of the quadratic function

\[
f(x) = 3x^2 + 7x + 4.
\]

The vertex is located:

   (A) Above the \( x \)-axis  (B) Below the \( x \)-axis
   (C) At the origin  (D) On the \( x \)-axis but not at the origin
   (E) None of these

15. Which of the following has two \( x \)-intercepts?

   (1) \( f(x) = x^2 - 40 \)  
   (2) \( g(x) = -x^2 + 60x - 903 \)
   (3) \( h(x) = -(x + 125)^2 + 1 \)

   (A) 1 only  (B) 2 and 3 only  (C) 2 only
   (D) 1 and 3 only  (E) None of these
16. For the years 1975 through 1990, the average price, \( p \) (in dollars per million British thermal units), of fuel used to generate electricity in the U.S. can be modeled by the function

\[
p(t) = -0.021t^2 + 0.50t - 1.04
\]

where \( t \) is time in years since 1970. Estimate the maximum average price, \( p \), of fuel used according to this model.

(A) $11.9 per million Btu  
(B) $1.94 per million Btu  
(C) $2.1 per billion Btu  
(D) $10.2 per million Btu  
(E) None of these

17. Consider the polynomial function below. Which of the following statement(s) is/are true about this polynomial?

(1) This polynomial could have degree 6  
(2) This polynomial could have degree 4  
(3) This polynomial could have degree 8

(A) 1 and 2 only  
(B) 1 and 3 only  
(C) 1 only  
(D) 2 only  
(E) 2 and 3 only
18. For which of the following functions does \( y \to \infty \) as \( x \to -\infty \)?

(1) \( y = 4x^3 - 3x \)
(2) \( y = 2x^4 - x + 10 \)
(3) \( y = -x^6 + 3x^5 - 2 \)

(A) None of them  (B) 2 only  (C) 2 and 3 only
(D) 1 and 3 only  (E) 3 only

19. For the function \( f(x) = -3(x - a)^5(x + b)(x - c)^2 \), where \( a, b \) and \( c \) are positive real numbers, which of the following is/are correct?

(1) \( f(x) \) is a polynomial of degree 5  
(2) \( f(x) \) has a zero at \( x = b \)
(3) \( f(x) \) has an \( x \)-intercept at \( (c, 0) \) 
(4) \( y \to -\infty \) as \( x \to -\infty \) and \( y \to -\infty \) as \( x \to \infty \)

(A) 1 and 2 only  (B) 2 and 4 only  (C) 1 and 3 only  (D) 3 and 4 only

20. Find the value of ‘\( b \)’ so that \( x + 2 \) is a factor of \( P(x) = x^4 - (b + 1)x^2 - 5bx - 9b \).

(A) \( b = -2 \)  (B) \( b = 4 \)  (C) \( b = 2 \)  (D) \( b = -4 \)  (E) None of these

21. Factor \( T(x) = x^3 + x^2 + 13x + 30 \). One of the factors is:

(A) \( x + 3 \)  (B) \( x - 5 \)  (C) \( x + 2 \)  (D) \( x - 10 \)  (E) None of these

22. If 3 is a zero of \( f(x) = 18x^3 - 111x^2 + 161x + 30 \), what are the other real zeros of \( f(x) \)? The SMALLEST real zero is:

(A) \(-\frac{16}{5}\)  (B) \(\frac{16}{5}\)  (C) \(-\frac{1}{6}\)  (D) \(-\frac{1}{5}\)  (E) 0
23. Find a polynomial of lowest degree that has \(-3, -\sqrt{2}, \text{ and } \sqrt{2}\) as its zeros.

(A) \(x^3 + 3x^2 - 2x - 6\)  (B) \(x^2 + (-3 - \sqrt{2})x - 6\)
(C) \(x^3 + 3x^2 - 4x - 12\)  (D) \(x^2 - 2\sqrt{2x} + 3\)
(E) None of these

24. Find the domain and range of the function \(H(x) = 3^{x^2} - 4\).

(A) Domain: \((-2, \infty)\)  Range: \((-4, \infty)\)
(B) Domain: \((-\infty, \infty)\)  Range: \((-4, \infty)\)
(C) Domain: \((-\infty, \infty)\)  Range: \((0, \infty)\)
(D) Domain: \((2, \infty)\)  Range: \((4, \infty)\)
(E) Domain: \((-2, \infty)\)  Range: \((4, \infty)\)

25. How much MORE money will you earn in an account that compounds interest continually than in an account that compounds interest quarterly if you invest $3000 for 7 years at an interest rate of 11%?

(A) $67.02  (B) $59.37  (C) $101.16  (D) $32.52  (E) None of these

26. Consider the functions \(f(x)\) and \(g(x)\) represented by the tables shown below.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
<th>(x)</th>
<th>(g(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Which, if either, of these functions is one-to-one?

(A) both \(f(x)\) and \(g(x)\)  (B) \(g(x)\) only  (C) \(f(x)\) only
(D) neither \(f(x)\) nor \(g(x)\)
27. If \( h(x) = 20x - 62 \), what is \( h(h^{-1}(6)) \)?

(A) 2  (B) 58  (C) –6  (D) 6  (E) None of these

28. Find \( R^{-1}(x) \) if \( R(x) = \frac{C}{3x - 1} \) (\( C \) is a real number)

(A) \( R^{-1}(x) = \frac{C}{3} x + C \)  (B) \( R^{-1}(x) = \frac{3x - 1}{C} \)

(C) \( R^{-1}(x) = \frac{C + x}{3x}, x \neq 0 \)  (D) \( R^{-1}(x) = \frac{C - 3x}{x}, x \neq 0 \)

(E) None of these

29. Suppose \( g(x) \) is the inverse of the function \( f(x) \). Which of the following tables would be correct for \( g(2x) \) if \( f(x) \) is given in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

(A) \[
\begin{array}{c|cccc}
  x & 1 & 3 & 5 & 7 \\
g(2x) & -4 & -2 & 0 & 2 \\
\end{array}
\]

(B) \[
\begin{array}{c|cccc}
  x & 1 & 3 & 5 & 7 & 9 \\
g(2x) & -1 & -\frac{1}{2} & 0 & \frac{1}{2} & 1 \\
\end{array}
\]

(C) \[
\begin{array}{c|cccccc}
  x & 2 & 6 & 10 & 14 & 18 \\
g(2x) & -2 & -1 & 0 & 1 & 2 \\
\end{array}
\]

(D) \[
\begin{array}{c|cccc}
  x & \frac{1}{2} & \frac{3}{2} & \frac{5}{2} & \frac{7}{2} & \frac{9}{2} \\
g(2x) & -2 & -1 & 0 & 1 & 2 \\
\end{array}
\]
30. Which of the following most resembles the graph of \( y = -\log_3(x - a) \), where \( a \) is a positive constant?

(A) 

(B) 

(C) 

(D) 

(E)
31. For the function \( R(x) = \log_2 x \), which of the following is/are correct?

(1) The domain is \([0, \infty)\) \hspace{1cm} (2) The range is \((-\infty, \infty)\) \hspace{1cm} (3) \(R(x)\) is one-to-one

(A) 2 and 3 only \hspace{1cm} (B) 1 and 2 only \hspace{1cm} (C) 3 only

(D) 1 and 3 only \hspace{1cm} (E) All of them

32. Express as a single logarithm and simplify if possible:

\[
\frac{1}{3} \log_a x + 4 \log_a y - 2 \log_a z
\]

(A) \( \log_a [x^{1/3} + y^4 - z^2] \) \hspace{1cm} (B) \( \log_a \left[ \frac{1}{3} x + 4 y - 2 z \right] \) \hspace{1cm} (C) \( \frac{7}{3} \log_a \left[ \frac{x y}{z} \right] \)

(D) \( \log_a \left[ \frac{x^{1/3} y^4}{z^2} \right] \) \hspace{1cm} (E) None of these

33. If \( M \) and \( N \) are positive, which of the following is/are correct?

(1) \( \ln(MN) = \ln M + \ln N \) \hspace{1cm} (2) \( \ln(M + N) = \ln M + \ln N \)

(3) \( \frac{\ln M}{\ln N} = \ln M - \ln N \)

(A) All of them \hspace{1cm} (B) None of them \hspace{1cm} (C) 1 only

(D) 1 and 3 only \hspace{1cm} (E) 2 and 3 only

34. Use natural logarithms to solve for \( x \) : \( 3 + 6e^{2x} = 5 \)

(A) \( x = \frac{1}{2} \ln 3 \) \hspace{1cm} (B) \( x = 2 \ln 3 \) \hspace{1cm} (C) \( x = \frac{1}{2} \ln \left( \frac{1}{3} \right) \)

(D) \( x = \ln 3 - 2 \) \hspace{1cm} (E) None of these
35. Solve for \( x \) : \( \log_3 x - \log_3(x - 1) = 2 \)

\[ \text{(A) } \frac{1}{8} \quad \text{(B) } \frac{9}{8} \quad \text{(C) } \frac{10}{9} \quad \text{(D) } \frac{100}{99} \quad \text{(E) None of these} \]

36. In 1980, the population of the United States was approximately 226.5 million people. In 1990, the population had grown to approximately 246.7 million. Assuming an exponential growth model \( A = Pe^{rt} \), what is the projected population of the U.S. in the year 2000?

(A) Less than 260 million
(B) Between 260 million and 265 million
(B) Between 265 million and 270 million
(B) Between 270 million and 275 million
(B) More than 275 million

37. A computer virus has been introduced into a system of computers. The data below was collected over the next 5 months:

<table>
<thead>
<tr>
<th>Number of computers infected</th>
<th>Months since virus was introduced</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>1</td>
</tr>
<tr>
<td>96</td>
<td>2</td>
</tr>
<tr>
<td>384</td>
<td>3</td>
</tr>
<tr>
<td>1536</td>
<td>4</td>
</tr>
<tr>
<td>6144</td>
<td>5</td>
</tr>
</tbody>
</table>

Which of the following equations best describes the number of computers infected, \( y \), as a function of the time, \( t \) (measured in months)?

(A) \( y = 6(4^t) \) \quad (B) \( y = 24(4^t) \) \quad (C) \( y = 24 + 72t \)
(D) \( y = 24 + t \) \quad (E) None of these
38. Based on the pattern of the data from the previous question, find the y-intercept of this function, and describe its practical significance.

(A) (0,6); there were no computers infected at time \( t = 6 \)
(B) (0,6); 6 computers were initially infected
(C) (0,4); there were no computers infected at time \( t = 4 \)
(D) (0,4); 4 computers were initially infected

39. True or False: The table given below represents \( y \) as a function of \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>3</th>
<th>5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>−1</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

(A) True (B) False

40. True or False: The graph of the function \( k(x) = x^2 - 24x + 159 \) has two \( x \)-intercepts.

(A) True (B) False

41. True or False: The graph of the function \( f(x) = 2^{x-5} + 3 \) has a horizontal asymptote at \( y = 3 \).

(A) True (B) False
FORMULAS

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ A = P e^{rt} \]