LaTeX EXERCISES

YOU

Abstract. Please recreate as much of this document as you can.

Contents

1. Problems

Problem 1. *If* $ax^2 + bx + c = 0$*, then*
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Problem 2. $1 \cdot 2 \cdots n = n! > \exp(n)$ *whenever* $n > 6$.

Problem 3. $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$.

Problem 4.
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Problem 5. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 2$.

Problem 6.
$$\sum_{i=1}^{n} i = \frac{1}{2}(n^2 + n).$$

Problem 7.
$$\begin{aligned} \mathrm{SL}(2, \mathbb{Z}) &= \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbb{Z}, ad - bc = 1 \right\}, \\ &= \left\langle \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right\rangle. \end{aligned}$$

Problem 8. *Q: How many elements are in a commutative group?*
A: Abelian.
Q: What's purple and commutes?
A: An Abelian grape.

2. Some Statements

Definition. A group G is **solvable** if there exists a chain of subgroups
$$\{e\} = G_n \lhd G_{n-1} \lhd \cdots \lhd G_1 \lhd G_0 = G$$
such that G_i/G_{i+1} is an abelian group for all $0 \leq i < n$.

Proposition 1. *If Γ is a Fuchsian group with compact orbit space $\mathbb{H}/\Gamma$ of genus g then there are elements $a_1, b_1, \ldots, a_g, b_g, c_1, c_2, \ldots, c_r$ in $\mathrm{Aut}(\mathbb{H})$ such that the following hold.*

(1) *We have $\Gamma = \langle a_1, b_1, \ldots, a_g, b_g, c_1, c_2, \ldots, c_r \rangle$.*
(2) *Defining relations for Γ are given by*

$$c_1^{m_i}, c_2^{m_2}, \ldots, c_r^{m_r}, \prod_{i=1}^{g} [\Phi(a_i), \Phi(b_i)] \prod_{i=1}^{r} \Phi(c_i)$$

Theorem 1 (Fundamental Theorem of the SP 2008 REU on Riemann Surfaces)**.** *A finite group G acts as a group of automorphisms of some compact Riemann surface of genus $g \geq 2$ if and only if G is isomorphic to Γ/K where Γ is a Fuchsian group with compact orbit space, and K is a Fuchsian surface group with orbit genus g that is a normal subgroup of Γ.*

Proposition 2 (Riemann-Hurwitz Formula for Orbit Spaces)**.** *Suppose Γ is a Fuchsian group with signature $(g; m_1, m_2, \ldots, m_r)$, and that the Riemann surface X is uniformized by a normal subgroup K of finite index with $\Gamma/K \cong G$. Then the genera of X and X/G, i.e., the orbit genera of K and Γ are related by the formula*

$$g(X) - 1 = |G|\Big(g(X/G) - 1\Big) + \frac{|G|}{2} \sum_{i=1}^{r} \left(1 - \frac{1}{m_i}\right).$$

Proof. See [Bre]. □

References

[Bre] Breuer, T. *Characters and Automorphism Groups of Compact Riemann Surfaces*. Cambridge, UK: Cambridge University Press, 2000.