

## Some GAP functions for dealing with finitely presented groups

- AbelianInvariants
- AbelianInvariantsSubgroupFpGroup
- IsomorphismPermGroup
- IsomorphismFpGroup
- GQuotients
- LowIndexSubgroupsFpGroup
- PQuotient
- EpimorphismNilpotentQuotient

Note that GAP distinguishes between **presentations** and **finitely presented groups**.

## Examples for Finitely Presented Groups

1. Consider the following GAP function.

```
SomeGroupAsFpGroup:= function( n )
  local F, gens, rels, i, j;

  # There are n-1 generators.
  F:= FreeGroup( n-1 );
  gens:= GeneratorsOfGroup( F );

  # The generators shall have order (at most) two.
  rels:= List( gens, s -> s^2 );

  # The product of adjacent generators shall have order (at most)
```

```

Append( rels, List( [ 1 .. n-2 ], i -> ( gens[i]*gens[i+1] )^3 )

# Non-adjacent generators shall commute.
for i in [ 1 .. n-1 ] do
  for j in [ i+2 .. n-1 ] do
    Add( rels, ( gens[i] * gens[j] )^2 );
  od;
od;

# This defines the group.
return F / rels;
end;

```

Ask **GAP** to compute the order of the groups for small values of  $n$ .

Theoretical question: Can you find –without using **GAP**– an epimorphism onto the symmetric group on  $n$  points, using suitable involutions as images of the generators?

2. The dihedral group of order  $2n$  can be written as

$$\langle a, b | a^2, b^2, (ab)^n \rangle.$$

Show –without using **GAP**– that this group is isomorphic with

$$\langle c, d | c^2, d^n, cd = d^{-1}c \rangle.$$

(Construct epimorphisms in both directions.)

3. Use **GAP** to show that the group

$$\langle a, b | a^2, b^2 \rangle$$

is infinite.

(Try `Size` and `IsFinite`; try `LowIndexSubgroupsFpGroup` and apply `AbelianInvariants` to the subgroups that are returned.)

Use the previous exercise to prove the claim without **GAP**.

4. Find out whether `FibonacciGroup( n )`, for  $1 \leq n \leq 6$ , is finite.  
(You can use the same idea as in the previous exercise.)

5. Consider the groups

$$\langle a, b \mid a^2, b^3, (ab)^n \rangle,$$

for  $2 \leq n \leq 7$ .

Which are finite, which are not? Does the idea used in the previous exercises work?

6. Compute the abelian invariants of `FibonacciGroup(5)` using `GAP`.

Write down the matrix of abelianized relators and compute its Smith normal form both using `GAP` and by hand.

(`GAP` functions `RelatorsOfFpGroup` and `SmithNormalFormIntegerMat`)