The traditional calculus sequence under mounting technological change

Seán Stewart

Core Mathematics, The Petroleum Institute
Abu Dhabi, United Arab Emirates

Learning Technologies and Mathematics Middle East Conference, 2007
Outline

1. Introduction
2. Background and context
3. Observations, reflections and challenges
4. Summary
Nowadays, affordable, hand-held, CAS-enabled graphics calculators are readily available.

Proponents for their adoption cite reasons such as:
- Their introduction is inevitable.
- Promotes renewed interest and motivation in the teaching and learning of calculus.
- Allows more interesting and realistic examples to be presented.

Detractors against their adoption cite reasons such as:
- Attention will shift from the simple to the sophisticated.
- Development of an over-dependence on the technology.
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And between these two opposing views lies me... 

This talk will outline my own experiences and perspectives in teaching a traditional calculus sequence with a TI-89 graphics calculator.
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First, a little background. . .

- The Petroleum Institute is a US-style tertiary level institution located in Abu Dhabi.
- It currently offers degrees in one of five engineering disciplines.
- Core Mathematics is one of a number of service departments.

All students take the following calculus sequence of courses.
- Calculus I (limits, differentiation and integration).
- Calculus II (infinite series, vectors and three-dimensional analytic geometry).
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Calculus is an area of mathematics where the procedural beautifully intermingles with the conceptual.

**Before** – Focused primarily on the teaching of pencil and paper algorithms, techniques and procedures.

**Now** – Focus shifts from procedural problem-solving tasks to more conceptually oriented tasks.
Observations, reflections and challenges

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The diagram below shows the graph of the gradient function $y = f'(x)$ for the curve $y = f(x)$.

For what value of $x$ does $y = f(x)$ have a local maximum? Justify your answer.
Challenges to a shift in emphasis

A shift to more conceptually oriented questions is not without its challenges.

Students tend to resist such questions for a number of reasons.

- Are unfamiliar (non-routine).
- Are not within a students notion of what real mathematics ought to be.
- They require the student to think!

Many students with good computational skills often perform poorly when their conceptual understanding is closely scrutinised.
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Other influences

Changes to the type of questions asked.

**Before** – Find, solve, evaluate, calculate, etc. Questions were largely procedural based.

**Now** – Sudden appearance of writing tasks! Allows the student to practice and develop articulating themselves in a mathematical context using English.
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- mathematical insight, and
- form recognition.

This can be successfully done within a CAS-enabled environment but one must be prepared to re-focus their teaching.
Continuing importance of by-hand techniques

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Example 2

To illustrate how one can re-focus their teaching, consider the following example.

If \( \int_{0}^{3} f(x) \, dx = 6 \), find \( \int_{0}^{1} f(3x) \, dx \).
A number of challenges present themselves when students are given unresisted access to a TI-89 graphics calculator.

- Weaker students use it as a crutch rather than a tool.
- All problems are approached using the button punching frenzy method rather than carefully thinking about it first.
- Students are not fully aware of the machine’s limitations.
- Inappropriate use.
- Tendency to accept calculator output blindly and uncritically.
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Example 3

A personal favourite of mine on student’s blindly accepting calculator output is the following.

Find \( \frac{d}{dx} \left( \frac{e^x + e^{-x}}{2} \right) \)

Answer 1: \( \frac{e^x - e^{-x}}{2} \) (obtained by hand)

Answer 2: sinh \( x \) (obtained using the calculator)

The question is asked before hyperbolic functions have been considered and student often ask from where did the constant ‘\( h \)’ come from!
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A common complaint is, ‘I teach mathematics, not how to operate a machine’.

Viewed as a cheating device rather than as a learning tool.

Perceived loss in pencil and paper mastery of essential techniques and skills.

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Frees up time students spend on performing by-hand manipulative tasks for more important and valuable learning tasks.

Provides a welcome relief in later courses.

Engineering students respond positively to their use particularly when used to achieve a desired outcome.

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Clearly, no one approach to teaching ought to be favoured over another.

My teaching with a TI-89 is more an extension of existing approaches rather than a dramatic shift to new and sophisticated content.

Finally, I see graphics calculator as but one of a number of epistemic tools available to the teacher to help aid student learning.
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Finally, I see graphics calculator as but one of a number of epistemic tools available to the teacher to help aid student learning.
...while getting an answer without knowing why may be interesting and useful, it is not mathematics.

B. Pender (2000)

Thank you!