Mathematical Writing

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- **Constant Scalar Curvature Metrics on Boundary Complexes of Cyclic Polytopes.** *With Daniel Champion, Andrew Marchese*, and Jacob Miller *.* Submitted

  In [7], a notion of constant scalar curvature metrics on piecewise flat manifolds is defined. Such metrics are candidates for canonical metrics on discrete manifolds. In this paper, we define a class of vertex transitive metrics on certain triangulations of $\mathbb{S}^3$; namely, the boundary complexes of cyclic polytopes. We use combinatorial properties of cyclic polytopes to show that, for any number of vertices, these metrics have constant scalar curvature.

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- **Ricci Yang-Mills Solitons on Nilpotent Lie Groups.** *With Michael Jablonski.* Submitted

  The purpose of this paper is to introduce the Ricci Yang-Mills soliton equations on nilpotent Lie groups N. We demonstrate that such metrics arise from automorphisms of N/Z, where Z is the center of N. Additionally, using techniques from Geometric Invariant Theory, we produce a characterization of symmetric Ricci Yang-Mills solitons on 2-step nilpotent Lie groups as critical points of a natural functional.

  Applying our work on nilpotent Lie groups, we study compact torus bundles over tori with locally (nilpotent) homogeneous metrics. On such spaces, we prove that Ricci Yang-Mills solitons are precisely the metrics whose Ricci tensor is invariant under the geodesic flow.

  We finish this note by producing examples of Lie groups that do not admit Ricci soliton metrics but that do admit Ricci Yang-Mills soliton metrics.

- **Regge's Einstein-Hilbert Functional on the Double Tetrahedron.** *With Daniel Champion and David Glickenstein.* To appear Diff. Geom. App.

The double tetrahedron is the triangulation of the three-sphere gotten by gluing together two congruent tetrahedra along their boundaries. As a piecewise flat manifold, its geometry is determined by its six edge lengths, giving a notion of a metric on the double tetrahedron. We study notions of Einstein metrics, constant scalar curvature metrics, and the Yamabe problem on the double tetrahedron, with some reference to the possibilities on a general piecewise flat manifold. The main tool is analysis of Regge's Einstein-Hilbert functional, a piecewise flat analogue of the Einstein-Hilbert (or total scalar curvature) functional on Riemannian manifolds. We study the Einstein-Hilbert-Regge functional on the space of metrics and on discrete conformal classes of metrics.

- **Cross Curvature Flow on a Negatively Curved Solid Torus.** *With Jason DeBlois and Dan Knopf.* Algebr. Geom. Topol. 10, 343-372, 2010.

  The classic 2π-Theorem of Gromov and Thurston constructs a negatively curved metric on certain 3-manifolds obtained by Dehn filling. By Geometrization, any such manifold admits a hyperbolic metric. We outline a program using cross curvature flow to construct a smooth one-parameter family of metrics between the "2π-metric" and the hyperbolic metric. We make partial progress in the program, proving long-time existence, preservation of negative sectional curvature, curvature bounds, and integral convergence to hyperbolic for the metrics under consideration.

- **Stability of the Ricci Yang-Mills Flow at Einstein Yang-Mills Metrics.** Comm. Anal. Geom. Vol 18, No. 1, 77-100, 2010.

  Let P be a principal $U(1)$-bundle over a closed manifold M. On P, one can define a modified version of the Ricci flow called the Ricci Yang-Mills flow, due to these equations being a coupling of Ricci flow and the Yang-Mills heat flow. We use maximal regularity theory and ideas of Simonett concerning the asymptotic behavior of abstract quasilinear parabolic partial differential equations to study the stability of the volume-normalized Ricci Yang-Mills flow at Einstein Yang-Mills metrics in dimension two. In certain cases, we show the presence of a center manifold of fixed points, while in others, we show the existence of an asymptotically stable fixed point.

- **Asymptotic Stability of the Cross Curvature Flow at a Hyperbolic Metric.** *With Dan Knopf.* Proc. Amer. Math. Soc. 137 (2009), 699-709.

  We show that for any hyperbolic metric on a closed 3-manifold, there exists a neighborhood such that every solution of a normalized cross curvature flow with initial data in this neighborhood exists for all time and converges to a constant-curvature metric. We demonstrate that the same technique proves an analogous result for Ricci flow. Additionally, we prove short-time existence and uniqueness of cross curvature flow under slightly weaker regularity hypotheses than was previously known.

- **Modified Ricci Flow on a Principal Bundle.** Ph.D. dissertation.

  Let M be a Riemannian manifold with metric g, and let P be a principal G-bundle over M having connection one-form a. One can define a modified version of the Ricci flow on P by fixing the size of the fiber. These equations are called the Ricci Yang-Mills flow, due to their coupling of the Ricci flow and the Yang-Mills heat flow. In this thesis, we derive the Ricci Yang-Mills flow and show that solutions exist for a short time and are unique. We study obstructions to the long-time existence of the flow and prove a compactness theorem for pointed solutions. We represent the Ricci Yang-Mills flow as a gradient flow and derive monotonicity formulas that can be used to study breather and soliton solutions. Finally, we use maximal regularity theory and ideas of Simonett concerning the asymptotic behavior of abstract quasilinear parabolic partial differential equations to study the stability of the Ricci Yang- Mills flow in dimension 2 at Einstein Yang-Mills metrics.