Math 105, Fall 2006 (Prof. Bayly) EXAM 3 (8 November 2006)

There are four (4) questions on this exam, but they are not the same length, difficulty, or point value. (I will be assigning a take-home component worth 10 more points.) You are not expected to complete everything, but you should do as much as you can.

(1)(25 points) Athens, Babylon, and Carthage are all hoping to be chosen as the site of the 200 BC Olympic Games. The Mediterranean Olympic Committee takes an unofficial vote and finds the following results: ABC(7), BCA(8), CAB(10), and ACB(4). (The numbers are the number of ballots with that preference order, with favorite on the left and least favorite on the right.)

(a)(3 points) Who would be the simple plurality winner?

(b)(10 points) Find the plurality-with-elimination winner.

After discussing the unofficial vote, the 4 folks who voted ACB decide they prefer Carthage, so in the official vote the ballots are ABC(7), BCA(8), and CAB(14).

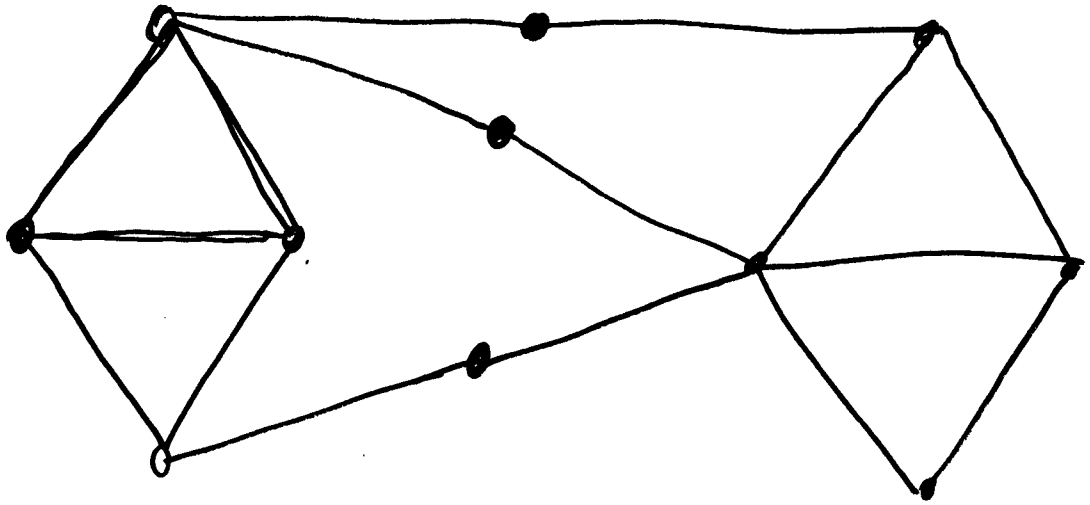
(c)(2 points) Who would be the simple plurality winner now?

(d)(10 points) Find the plurality-with-elimination winner. What's surprising about this result?

(2)(20 points) (a)(5 points) The graph/network below has no Euler path or circuit. Why?

(b)(10 points) Copy the graph onto your test paper, and add enough duplicates of existing edges so that the graph now has an Euler *path*. What is the path?

(c)(5 points) What edge duplicates could you add to make the graph from part (b) have an Euler *circuit*? (You do NOT have to find it!)



(3)(20 points) The editors of *Gourmet* magazine rank the restaurants Andre's, Borrelli, Casablanca, Dante, and Escargot. The preference order tallies are ABCDE(8), DBACE(7), DBECA(6), CABDE(2), and EADBC(1).

(a)(5 points) Who is the majority winner?

(b)(15 points) Who is the Borda count winner, if 1st place is worth 5 points, 2nd = 4 points, 3rd = 3, 4th = 2, and 5th = 1?

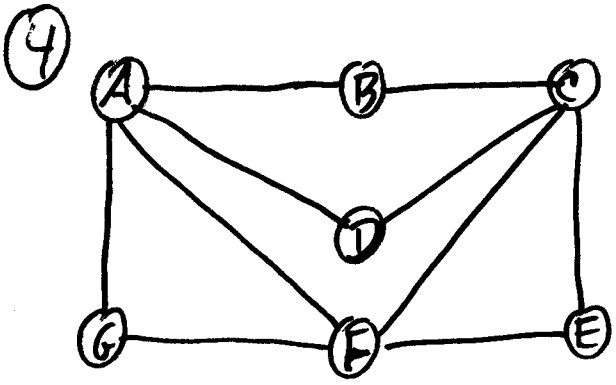
(4)(10 points) For the network on the left, below, find a Hamiltonian path, and explain why there is no Hamiltonian circuit.

(5)(15 points) The network on the right, below, represents travel times between branches of a delivery service. The driver wants to know a round-trip order to visit the branches that takes the smallest total time.

(a)(5 points) Use the *cheapest link* method to find a not-too-unreasonable route for the driver to take.

(b)(5 points) Use the *nearest neighbor* method starting at A, to find a not-too-unreasonable route.

(c)(5 points) Describe a method *guaranteed* to find the very shortest round-trip route.



Network for problem (4)

(5) Network for problem (5)