[Thanks to Matt Johnson for making up these solutions, and Justin Spargur for computer typesetting!]

For self-grading, problems 1.4(2), 1.4(14), and 1.6(13) are each worth 2 points; each of the others is worth 1 point, making a maximum of 10 points. I don't care if you give yourself partial credit in fractions of a point, but please arrange the total to be an integer! Write the total prominently on the front at the top of the page, and circle it so we can't possibly miss it. Thanks - Prof. Bayly

Section 1.4

2. a) $$\begin{bmatrix} 4 & 1 \\ 5 & 1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 4+3 \\ 5+3 \\ 6+3 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

$$\max \begin{bmatrix} 4 & 1 \\ 5 & 1 \\ 6 & 1 \end{bmatrix} = 7 \qquad \max \begin{bmatrix} 1 \\ 3 \end{bmatrix} = 3 \qquad \max \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = 9$$

and observe $7 \cdot 3 \geq 9$

b) $$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$$

$$\max \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = 24 \qquad \max \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 1 \qquad \max \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} = 8$$

observe $24 \cdot 1 \geq 8$

c) $$\begin{bmatrix} 4 & 3 \\ 6 & 6 \\ 8 & 9 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{4}{2} + \frac{3}{3} \\ \frac{6}{2} + \frac{6}{3} \\ \frac{8}{2} + \frac{9}{3} \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$$

$$\max \begin{bmatrix} 4 & 3 \\ 6 & 6 \\ 8 & 9 \end{bmatrix} = 17 \qquad \max \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \end{bmatrix} = \frac{1}{2} \qquad \max \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} = 7$$

observe $17 \cdot \frac{1}{2} \geq 7$

4. A is an $m \times n$ matrix , x is an n-dimensional matrix.

$$Ax \longrightarrow m\left\{\left[\overbrace{\qquad\qquad}^{n}\right.\right. \times \left.\left[\quad\right]\right\}n$$

m operations of a $1 \times n$ times an $n \times 1$
so $m \cdot n$ separate multiplications

B is an $n \times p$ matrix $\quad m\left\{\left[\overbrace{\qquad\qquad}^{n}\right.\right. \times \; n\left\{\left[\overbrace{\qquad\qquad}^{p}\right.\right.$

$AB \nearrow$

to multiply A by B we perform the first $A\times$ multiplication
p times, so we get $mn \cdot p$ separate multiplications.

14. a) $A^2 = -I$ $\quad A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

b) $B^2 = 0,\ B \neq 0$ $\quad B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

c) $CD = -DC, \quad CD \neq 0$ $\quad C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

d) $EF = 0$, no entries of E or F are zero. $\quad E = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad F = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

19. Which of the following are equal to $(A+B)^2$

$(B+A)^2, \quad A^2 + 2AB + B^2, \quad A(A+B) + B(A+B), \quad (A+B)(B+A), \quad A^2 + AB + BA + B^2$

All except $A^2 + 2AB + B^2$ because $2AB \neq AB + BA$ (matrix multiplication is <u>not</u> commutative.

23. For $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad C = AB = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

find $A^n,\ B^n,\ C^n$, for all integers n (positive).

$A^2 = A$, so $A^3 = A \cdot A = A$, thus $A^n = A$

$B^2 = I$, so $B^3 = BI = B$, so $B^n = B$ if n is odd, $B^n = I$ if n is even.

$C^2 = 0$, so $C^n = 0$ if $n > 1$

Section 1.6

13. $A = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ $\qquad ||A|| = 3 \ \ ||A^T|| = 4 \ \ ||B|| = 2 \ \ ||B^T|| = 4$

$A^T B = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \end{bmatrix}$ $\qquad ||A^T B|| = 8 \leq 4 \cdot 2$

$B^T A = \begin{bmatrix} 2 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \end{bmatrix}$ $\qquad ||B^T A|| = 8 \leq 4 \cdot 3$

$AB^T = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 2 & 2 \end{bmatrix}$ $\qquad ||AB^T|| = 12 \leq 4 \cdot 3$

$BA^T = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 6 & 2 \end{bmatrix}$ $\qquad ||BA^T|| = 8 \leq 2 \cdot 4$

15. For a square matrix B, $A = B + B^T$ is symmetric, proof

$A_{ij} = B_{ij} + B_{ji}$, and $A_{ji} = B_{ji} + B_{ij} = A_{ij}$

since $A_{ij} = A_{ji}$ for all i, j, $A^T = A$.

$K = B - B^T$ in skew symmetric

$K_{ij} = B_{ij} - B_{ji},\ K_{ji} = B_{ji} - B_{ij}$, so $K_{ij} = -K_{ji}$ for all i, j

Thus, $K^T = -K$

for $B = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}, \quad K = \begin{bmatrix} 0 & 0 \\ -2 & 0 \end{bmatrix}$

$B = \frac{1}{2}A + \frac{1}{2}K$