to Justin Spargur
for Math 400 (LaTex)
Thanks, Matt Johnson

section 1.3

(6) $\begin{array}{l} au + v = 1 \\ 4u + av = 2 \end{array} \Rightarrow \left[\begin{array}{cc|c} a & 1 & 1 \\ 4 & a & 2 \end{array}\right]$

$\Rightarrow \begin{bmatrix} 1 & 1/a & 1/a \\ 4 & a & 2 \end{bmatrix}$ so ~~a≠0~~ $a = 0$ corresponds to a break down

$\Rightarrow \begin{bmatrix} 1 & 1/a & 1/a \\ 0 & a - 4/a & 2 - 4/a \end{bmatrix}$ so consider when $a - 4/a = 0$

$a = \pm 2$

~~so~~

$a = 2$ infinite number of solutions
$a = -2$ zero solutions
$|a| \neq 2$ one solution

section 1.5

(5) $Ax = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}$

then $Ux = c$ is $\begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$

and $A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix}$

section 2.2

(6)
$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -2v-3 \\ v \\ 2 \end{bmatrix} = v \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$$

let $A = \begin{bmatrix} -2v-3 \\ v \\ 2 \end{bmatrix}$ then $\|A\| = \left((-2v-3)^2 + v^2 + 2^2 \right)^{1/2}$

$$= \left(5v^2 + 12v + 13 \right)^{1/2}$$

solve $10v + 12 = 0$, $v = -6/5$

so smallest solution is $\begin{bmatrix} -3/5 \\ -6/5 \\ 2 \end{bmatrix}$

(8)
$$\begin{aligned} u + v + 2w &= 2 \\ 2u + 3v - w &= 5 \\ 3u + 4v + w &= c \end{aligned} \qquad \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 3 & -1 & 5 \\ 3 & 4 & 1 & c \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & -5 & 1 \\ 0 & 1 & -5 & c-6 \end{bmatrix}$$

$c - 6 = 1$, $c = 7$
$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & -5 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 7 & 1 \\ 0 & 1 & -5 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_2 - 5x_3 = 1 \qquad x_1 + 7x_3 = 1$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \to \begin{bmatrix} 1 - 7x_3 \\ 1 + 5x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -7 \\ 5 \\ 1 \end{bmatrix}$$

$\min \left((1-7x_3)^2 + (1+5x_3)^2 + x_3^2 \right)^{1/2}$

$\min \left(75x_3^2 - 4x_3 + 2 \right)$

set $150x_3 - 4 = 0$

$x_3 = 2/75$

(19) $A = \begin{bmatrix} 1 & 2 & 0 \\ a & 8 & 3 \\ 0 & b & 5 \end{bmatrix}$ $a = 4$ leads to a row exchange
$b = 40/3 - 10a/3$ leads to a singular matrix

$A = \begin{bmatrix} c & 2 \\ 6 & 4 \end{bmatrix}$ $c = 0$ leads to a row exchange
$c = 3$ leads to a singular matrix

section 1.6

(6) $A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $A_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

$A_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ $A_2^{-1} = \begin{bmatrix} 3/4 & 1/2 & 1/4 \\ 1/2 & 1 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{bmatrix}$

$A_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $A_3^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(11) i) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

ii) $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

iii) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$

which we assumed exists

let $C = B^{-1} + A^{-1}$, $D = (A+B)^{-1}$, then $ACB = (A+B)$, so $ACBD = \underline{I}$

(9) $Ax = b$ $\quad A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 0 & 7 \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 0 & 3 & b_1 \\ 2 & 4 & 0 & 7 & b_2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 & 3 & b_1 \\ 0 & 0 & 0 & 1 & b_2 - 2b_1 \end{bmatrix}$$

$x_4 = b_2 - 2b_1$

$x_1 + 2x_2 + 3x_4 = b_1$, $\quad x_1 + 2x_2 = 7b_1 - 3b_2$

$x_1 = 7b_1 - 3b_2 - 2x_2$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \to \begin{bmatrix} 7b_1 - 3b_2 - 2x_2 \\ x_2 \\ x_3 \\ b_2 - 2b_1 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 7b_1 - 3b_2 \\ 0 \\ 0 \\ b_2 - 2b_1 \end{bmatrix}$$

$\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ are in the nullspace of A