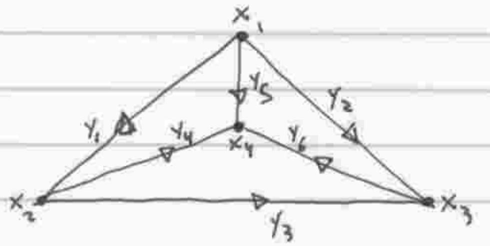
2.5.6



incidence matrix $A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

each of the smaller triangles forms a loop $y = \begin{array}{l} (1,0,0,1,-1,0) \\ (0,0,1,-1,0,1) \\ (0,1,0,0,-1,1) \end{array}$

2.5.8 $A^T C A = \begin{bmatrix} c_1+c_2+c_5 & -c_1 & -c_2 & -c_5 \\ -c_1 & c_1+c_3+c_4 & -c_3 & -c_4 \\ -c_2 & -c_3 & c_2+c_3+c_6 & -c_6 \\ -c_5 & -c_4 & -c_6 & c_4+c_5+c_6 \end{bmatrix}$

setting each $c_i = 1$ we get $A^T I A = A^T A = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$

pattern for diagonal of $A^T C A$ is entry in nth row is sum of c_i's for all i's that are connected to nth node

any c_i will appear in row j if i corresponds to an edge connected to the node j

2.4.19 $A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$

$N(A)$ has basis $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$

$R(A^T)$ has basis $\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} \right\}$

$N(A^T)$ has basis $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$

$R(A)$ has basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} \right\}$

3.4.6 find x, y, z so that $Q = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{14} & x \\ 1/\sqrt{3} & 2/\sqrt{14} & y \\ 1/\sqrt{3} & -3/\sqrt{14} & z \end{bmatrix}$ is orthogonal

we know $Q Q^T = I$, so we get

$$\tfrac{1}{3} + \tfrac{1}{14} + x^2 = 1 \qquad \text{so } |x| = 5/\sqrt{42}$$
$$\tfrac{1}{3} + \tfrac{4}{14} + y^2 = 1 \qquad |y| = 4/\sqrt{42}$$
$$\tfrac{1}{3} + \tfrac{9}{14} + z^2 = 1 \qquad |z| = 1/\sqrt{42}$$

since $x/\sqrt{3} + y/\sqrt{3} + z/\sqrt{3} = 0$ we see

$$x = -5/\sqrt{42}$$
$$y = 4/\sqrt{42}$$
$$z = 1/\sqrt{42}$$

only freedom is the sign, instead of $[x, y, z]$ we could use $[-x, -y, -z]$

observe $Q^T Q = I$, this implies rows are also orthonormal

3.4.14 $\quad a = (1,1,0) \quad b = (1,0,1) \quad c = (0,1,1)$

$$a'' = a/\sqrt{2} = (1/\sqrt{2}, 1/\sqrt{2}, 0)$$

$$b' = b - (a''^T b)\,a'' = (1,0,1) - (1/2, 1/2, 0) = (1/2, -1/2, 1)$$
$$b'' = b'/\|b'\| = (1/\sqrt{6}, -1/\sqrt{6}, \sqrt{2}/\sqrt{3})$$

$$c' = c - (a''^T c)\,a'' - (b''^T c)\,b'' = (0,1,1) - 1/\sqrt{2}\,(1/\sqrt{2}, 1/\sqrt{2}, 0) - 1/\sqrt{6}\,(1/\sqrt{6}, -1/\sqrt{6}, \sqrt{2}/\sqrt{3})$$
$$= (0,1,1) - \tfrac{1}{2}(1,1,0) - \tfrac{1}{6}(1,-1,2)$$
$$= (-2/3, 2/3, 2/3)$$

$$c'' = c'/\|c'\| = \sqrt{3}/2\,(-2/3, 2/3, 2/3) = (-1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$$

so $q_1 = (1/\sqrt{2}, 1/\sqrt{2}, 0) \quad q_2 = (1/\sqrt{6}, -1/\sqrt{6}, 2/\sqrt{6}) \quad q_3 = (-1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$

3.4.16 $a_1 = (1,2,2)$ $a_2 = (1,3,1)$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 2 & 1 \end{bmatrix} \quad Q = \begin{bmatrix} 1/3 & 0 \\ 2/3 & 1/\sqrt{2} \\ 2/3 & -1/\sqrt{2} \end{bmatrix} \quad R = \begin{bmatrix} 3 & 3 \\ 0 & \sqrt{2} \end{bmatrix}$$

A is $m\times n$, Q is $m \times n$, R is $n \times n$

4.2.1 How are $\det(2A)$, $\det(-A)$, $\det(A^2)$ related to $\det(A)$, A is $n\times n$

$$\det(2A) = 2^n \det(A), \quad \det(-A) = (-1)^n \det(A), \quad \det(A^2) = (\det(A))^2$$

4.2.10 $\det\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$

$$\frac{c-a}{b-a}\left(0, b-a, (b-a)(b+a)\right) = \left(0, 0, (c-a)(b+a)\right)$$

$$\begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & 0 & (c-a)(c-b) \end{vmatrix} = 1(b-a)(c-a)(c-b)$$

4.3.5 $\quad D_n = \det\begin{bmatrix}1 & -1 & & & \\ 1 & 1 & -1 & & \\ & 1 & 1 & -1 & \\ & & & \ddots & \\ & & & 1 & 1\end{bmatrix}$

$$D_n = \begin{vmatrix} & & & \\ 1 & -1 & & \\ 1 & 1 & -1 & \\ & & \ddots & \\ & & 1 & 1\end{vmatrix} + \begin{vmatrix} & & & \\ 1 & -1 & & \\ & 1 & -1 & \\ & & \ddots & \\ & & 1 & 1\end{vmatrix}$$

↑ this is just D_{n-1} $\qquad$ ↙ expand along row 1

so $D_n = D_{n-1} + \begin{vmatrix}1 & -1 & & \\ & 1 & -1 & \\ & & \ddots & \\ & & 1 & 1\end{vmatrix} = D_{n-1} + \begin{vmatrix} & & & \\ 1 & -1 & & \\ 1 & 1 & -1 & \\ & & \ddots & \\ & & 1 & 1\end{vmatrix} = D_{n-1} + D_{n-2}$