5.1.4 $du/dt = Pu$ $du/dt = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ $u_0 = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

then $u = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + 4 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^t$

5.1.7 λ is an eigenvalue of A, $Ax = \lambda x$

a) $B = A - 7I$, then $Bx = (A-7I)x = Ax - 7x = \lambda x - 7x$
$= (\lambda - 7)x$, so the eigenvalue is $\lambda - 7$

b) $Ax = \lambda x$ $(\lambda \neq 0)$
$A^{-1}Ax = A^{-1}\lambda x$
$x = \lambda A^{-1}x$ $1/\lambda \, x = A^{-1}x$, so $1/\lambda$ is the eigenvalue

5.1.14 $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

rank of A is 1
$\lambda = 0, 0, 0, 4$
$x = (1,1,1,1)$, $Ax = 4x$

rank of C is 2
$\lambda = 0, 0, 2, -2$
$x = (1,1,1,1)$ $Ax = 2x$
$y = (1,-1,1,-1)$ $Ay = -2y$

5.2.2 $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{bmatrix} -5 & 10 \\ -3 & 10 \end{bmatrix}$

5.2.3 $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $\lambda = 0, 0, 3$

$x_3 = (1, 1, 1)$

$\left.\begin{aligned} x_1 &= (1, 1, -1) \\ x_2 &= (1, 2, 0) \end{aligned}\right\}$ arbitrary vectors in the nullspace of A

$(1, 1, -2)$

$(-3, 2, 1)$ in the nullspace of A

$S_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix}$ $S_2 = \begin{bmatrix} 1 & 1 & -3 \\ 1 & 1 & 2 \\ 1 & -2 & 1 \end{bmatrix}$

5.2.6 a) if $A^2 = I$, then if x is such that $Ax = \lambda x$

$AAx = A\lambda x \Rightarrow Ix = A\lambda x$

$x = \lambda(Ax)$ since $\lambda \neq 0$ (because then $x = 0$)

$1/\lambda\, x = Ax$

so $1/\lambda = \lambda$, $\lambda = \pm 1$

b) if A is 2×2 $A \neq I, -I$, trace$(A) = 0$, det$(A) = -1$

c) if first row is $(3, -1)$, second row is $(8, -3)$

5.2.8 a) $\overset{n \times n}{A} = uv^T$, $Au = uv^Tu$, so let $\lambda = v^Tu$

then $A = \lambda u$

b) other eigenvalues must be zero because $\dim(N(A)) = n-1$

c) trace$(A) = v^Tu$

5.2.13 $A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ $\lambda = 1, 9$

$x_1 = (1, -1)$

$x_2 = (1, 1)$

$A = \underbrace{\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}}_{S^{-1}} \begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}_{S}$

find R, $R^2 = A$, how many square roots

square roots have the form $S^{-1}BS$, because then $(S^{-1}BS)(S^{-1}BS) = S^{-1}(B^2)S$

so $B^2 = \begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -3 & 0 \\ 0 & -1 \end{bmatrix}$

5.2.14 if A is diagonalizable, show that det of $A = S\Lambda S^{-1}$ is a product of its eigenvalues.

since S is invertible $\det(S) = 1/\det(S^{-1})$

so by property of det, the $\det(\Lambda)$ = product of each entry on the diagonal, which are the eigenvalues of A

5.3.1 for the fibonacci matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ write down A^2, A^3, A^4

and then A^{100}, $A^2 = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$, $A^3 = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$, $A^4 = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

then $A^{100} = \begin{bmatrix} F_{102} & F_{101} \\ F_{101} & F_{100} \end{bmatrix}$ where F_n is the n^{th} fibonacci number.

b) find B^{-101} if $B = \begin{bmatrix} 7 & 12 \\ -4 & -7 \end{bmatrix}$, we can write $B = \begin{bmatrix} -3/2 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -2 & -3 \end{bmatrix}$

so $B^{-101} = \begin{bmatrix} -3/2 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}^{-101} \begin{bmatrix} 2 & 4 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ -4 & -7 \end{bmatrix} = B$