5.6 solve for $\frac{du}{dt} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} u$ , $u_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ , $u_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\lambda_1 = 2 \quad (1,-1)$

$\lambda_2 = 4 \quad (1,1)$

$u_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} c_1 e^0 + c_2 e^0 \\ c_1 e^0 - c_2 e^0 \end{bmatrix}$ $\quad c_1 = 1/2, \; c_2 = 1/2$

$u(t) = 1/2 e^{2t} x_1 + 1/2 e^{4t} x_2$

$u_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 + c_2 \\ c_1 - c_2 \end{bmatrix}$ $\quad c_1 = 1/2, \; c_2 = -1/2$

$u(t) = 1/2 e^{2t} x_1 - 1/2 e^{4t} x_2$

$e^{At} = \frac{1}{2} \begin{bmatrix} e^{-2t} + e^{-4t} & e^{-2t} - e^{-4t} \\ e^{-2t} + e^{-4t} & e^{-2t} + e^{-4t} \end{bmatrix}$

5.10 $\frac{du}{dt} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} u$

$u(t) = c_1 e^{-\sqrt{2}it} x_1 + c_2 e^{\sqrt{2}it} x_2 + c_3 e^{0t}$

5.11 if $P$ is a projection matrix then $P^2 = P$
so if $x$ is in $S$, there's a $y$ so that $P^2 y = x$ and $Py = x$
$\Rightarrow P(Py) = x$, $\Rightarrow P(x) = x$, so $x$ is an eigenvector

5.21 $M = \begin{bmatrix} d & 0 & 0 \\ 0 & d^2 & 0 \\ 0 & 0 & d^3 \end{bmatrix}$ $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$M^{-1}AM = \begin{bmatrix} 1 & d & d^2 \\ 1/d & 1 & d \\ 1/d^2 & 1/d & 1 \end{bmatrix}$ assuming $d \neq 0$

eigenvalues are $0, 0, 3$

5.23 if $Ax = \lambda_1 x$, $A^T y = \lambda_2 y$, show $x^T y = 0$

we know $(Ax)^T = x^T A^T = \lambda_1 x^T$
so $A^T y = \lambda_2 y$, $\Rightarrow (x^T A^T) y = \lambda_2 x^T y$
$\Rightarrow \lambda_1 x^T y = \lambda_2 x^T y$, $\lambda_1 \neq \lambda_2$, so $x^T y = 0$

5.28 $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & d & c \\ 0 & 5 & 3 \end{bmatrix}$ if $A$ is hermitian, and has 3 diff eigenvalues
so $c = 5$
then $d$ must be chosen so that
the matrix $A$ has 3 distinct eigenvalues

5.30 $k \to \infty$ $\begin{bmatrix} .4 & .3 \\ .6 & .7 \end{bmatrix}^k \begin{bmatrix} 9 \\ 6 \end{bmatrix} \to \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$

p451 ① a) $A = [1, 1, 1, 1]$

$$[1]\,[4^{1/2}\ 0\ 0\ 0]\begin{bmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 0 & 0 & -1/2 & 1/2 \\ 0 & -1/2 & 0 & 1/2 \\ -1/2 & 0 & 0 & 1/2 \end{bmatrix}$$

$$A^+ = \begin{bmatrix} 1/2 & 0 & 0 & -1/2 \\ 1/2 & 0 & -1/2 & 0 \\ 1/2 & -1/2 & 0 & 0 \\ 1/2 & 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1/2 \\ 0 \\ 0 \\ 0 \end{bmatrix}[1]$$

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^+ = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A.2 $\quad A = [1,1,1,1]$

$$A = [1]\begin{bmatrix} 2 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ 1/2 & -1/2 & -1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 & -1/2 \end{bmatrix}$$

$$A^+ = \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^+ = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$C^+ = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix}$$

A.4 $\quad A = \frac{1}{\sqrt{10}} \begin{bmatrix} 10 & 6 \\ 0 & 8 \end{bmatrix}$

$$A^{+}A = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix} \qquad \begin{matrix} \lambda_1 = 16 & (1,1) \\ \lambda_2 = 4 & (1,-1) \end{matrix}$$

$$S = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

A.5 $\quad$ what is minimum length squares solution to $x^{+} = A^{+}b$

$$Ax = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$

$$x^{+} = (1, 1/2, 1/2)$$