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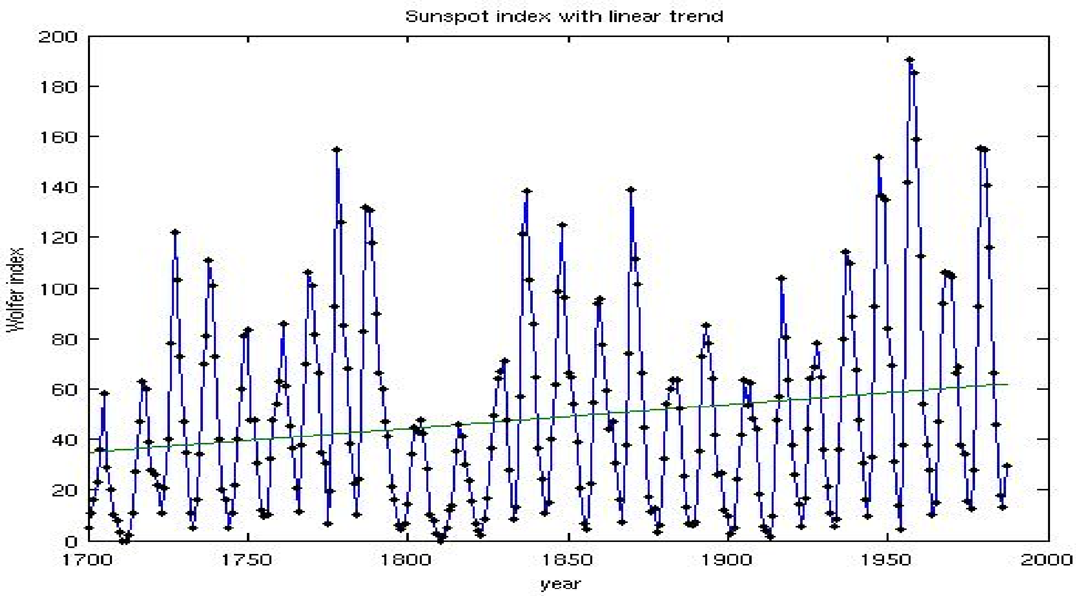
Summer 2009 VIGRE Program in Computational Photonics

I. Introduction to Plasma Generation and Wave Mixing

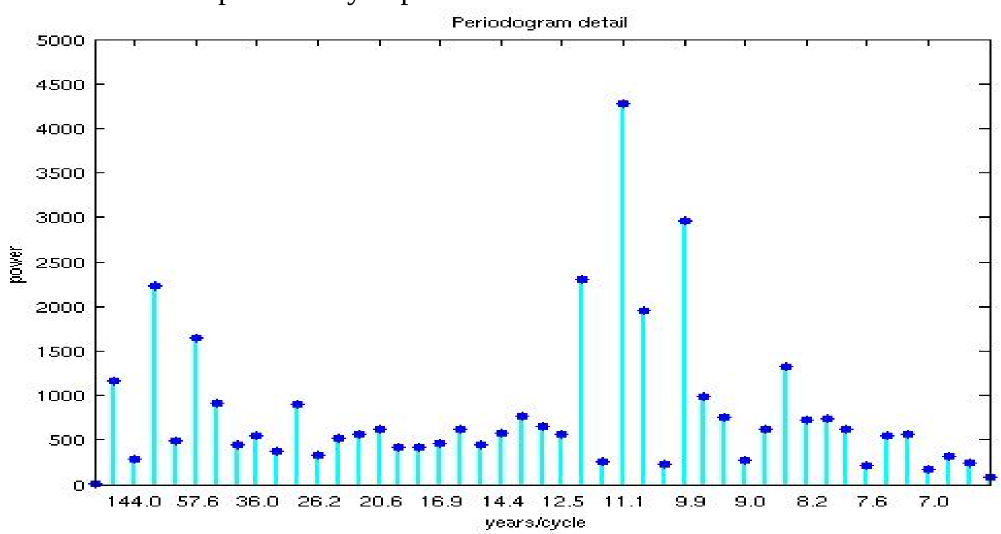
In the first week, I was introduced to the project's background by Moysey Brio, Miro Kolesik, and John Pate. I was also given past presentations given by John to examine. These presentations can be currently found on John's webpage at http://math.arizona.edu/~jpate/. After discussing my background knowledge, Moysey and John decided I would be best aided by investigating partial differential equations and Fourier Series for two weeks before tackling the project itself head on.

The rest of the first two weeks was dedicated to learning about elementary partial differential equations, including the vibrating string equations for the particle on a string, coupled oscillators, and continuous string problem. In this process I learned about the construction of the problem, the underlying parameters, and governing equations for each consideration. Also, I examined specific faults in underlying assumptions made about each problem; chief among them in all problems was Hooke's law, which states that force is linearly dependent on displacement. This is known to be true at only small displacement: when larger displacements occur the resulting relation is nonlinear in nature. Another assumption worth examining is that strings have uniformly linear mass. This obviously falls short in nature, as there are at least small differences in the mass of a string over space.

Another topic I examined during the first two weeks was Fourier Series. A Fourier Series decomposes a function into a sum of oscillating functions such as sine and cosine functions. Joseph Fourier created this method in attempting to solve the heat equations, which at the time was unsolvable. Since its rise to prominence the Fourier Series continues to be crucial in fields including signal and image processing, acoustics, and electrical engineering. I utilized an FFT (fast Fourier transform) to show different periodic behavior in sun spots given sun spot data from the past three hundred years. The code to run the program can be ascertained through "Numerical Computing with Matlab" by Cleve Moler.

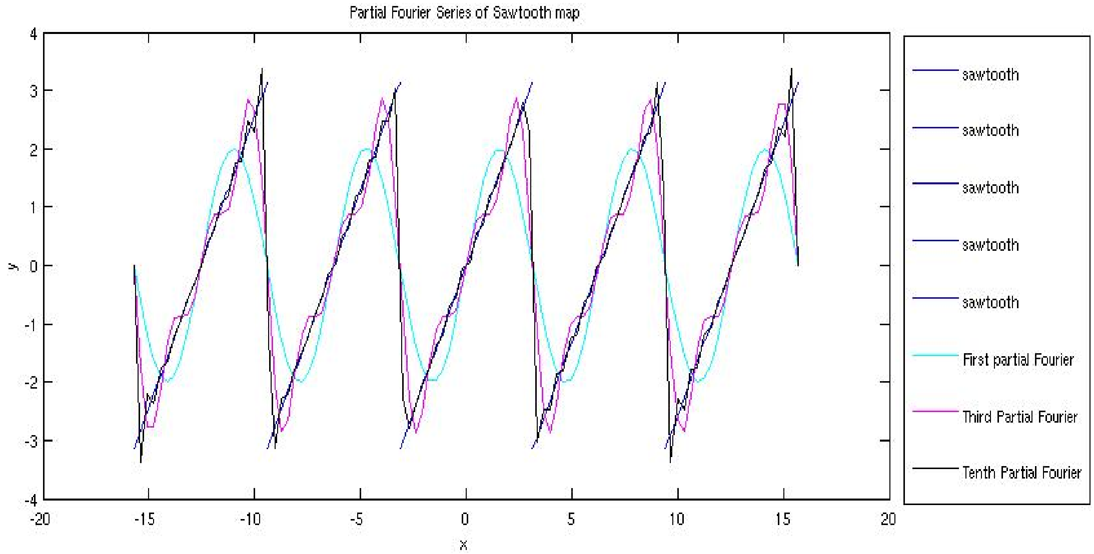


Here the data of sunspot activity is plotted and a linear trend is fitted.



Through use of an FFT we can show the periodic behavior of the data. It is important to note that although the greatest periodic behavior of sunspots is eleven years per cycle this is not the only periodic behavior it exhibits. Ten years per cycle, twelve years per cycle, and fifty-seven years per cycle are among other periodic cycles.

I then made my own Matlab plot for a sawtooth map to show the increasing accuracy of a Fourier Series as the partial Fourier number increases.

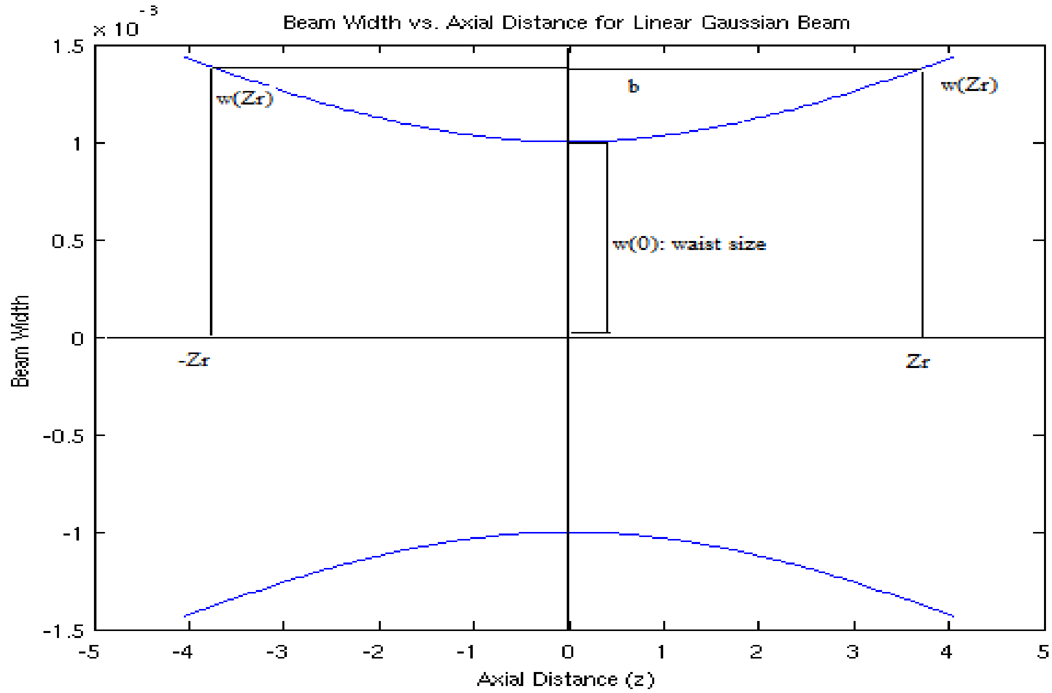


It is easy to see the increased accuracy of Fourier Series for a sawtooth map (y=x for period of 2*pi) as the partial Fourier increases.

In the remaining part of the first two weeks, I investigated more relevant partial differential equations to optics like the Paraxial approximation to the Helmholtz Equation and the Schrodinger Equation.

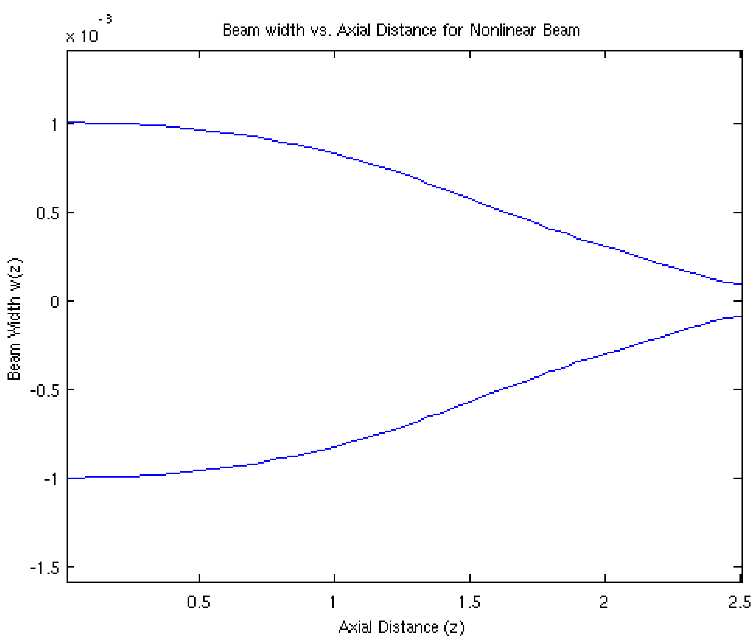
In the third and fourth weeks I was able to start running the program Miro wrote for laser simulation.

The first case I examined was the linear case for laser propagation. I studied the differing parameters that govern the beams and examined linear beam propagation in reference to these governing parameters.



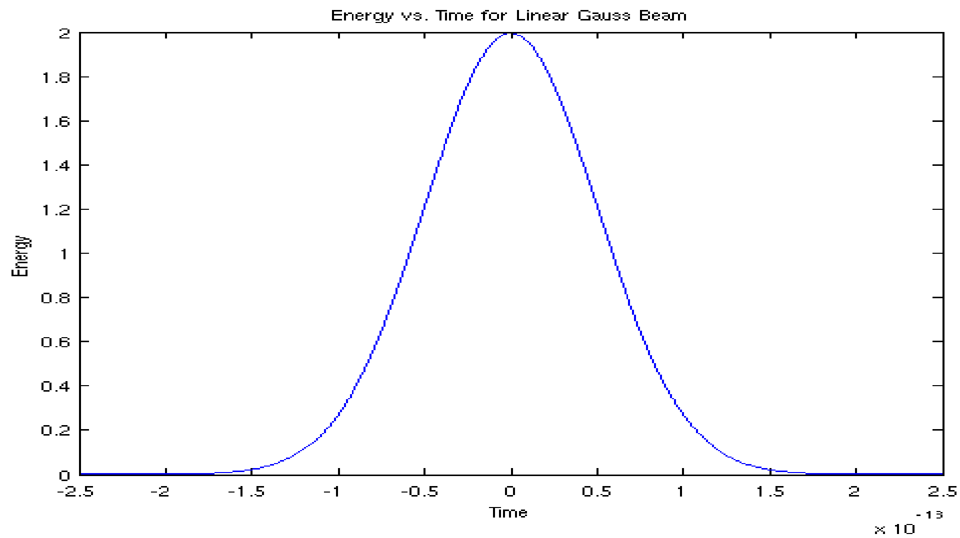
Here is a Gaussian beam propagating through space. The vertical axis represents the radial distance of the beam and the horizontal axis relates the space the beam is traveling through. It is immediately clear that the function relating the two axes is a Gaussian function. Upon further investigation it can be seen that this plot is closely governed by the beam parameters previously mention.

The second case I examined was the nonlinear case. Here the beam focuses on itself, thus decreasing its radial distance over time. This is contrary to the linear beam which defocuses and increases radial distance over space as seen above.



Here we can see the nonlinear beam focusing on itself and decreasing radial size over space. This self focus is usually indicative of Kerr-nonlinearity. It can be due to an increased refractive index in the inner part of the beam created by a larger optical intensity on the beam axis compared with the spatial intensity distribution. This modified refractive index then acts as a focusing lens. When a beam experiences this self-focusing phenomenon it is also liable to experience total collapse; a phenomenon where the optical medium may be damaged due to exceedingly high optical intensities.

Another graph worth looking at is the Energy vs. Time graph.

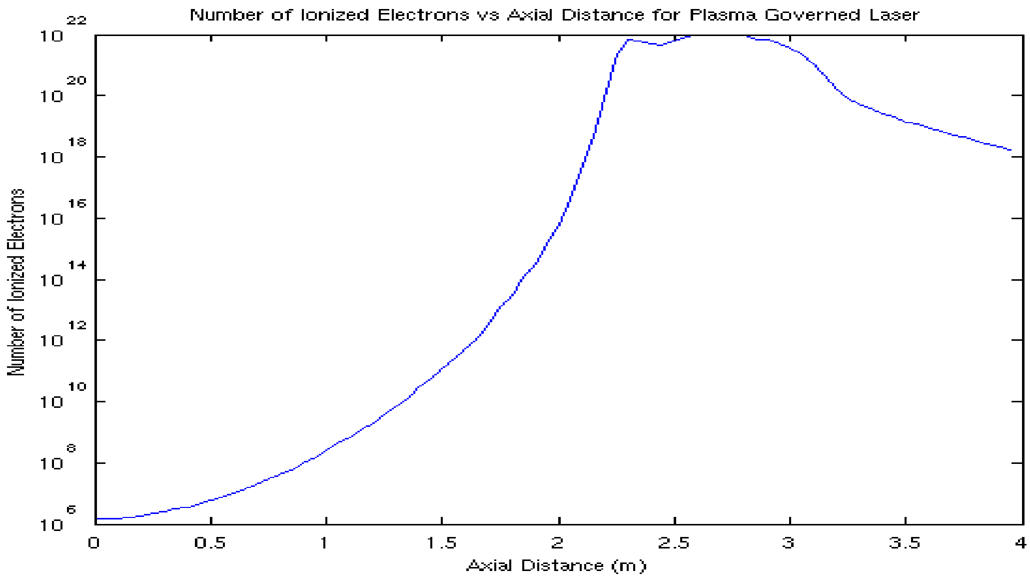


Here we see that a specific pulse occurs in around 300 femtoseconds.

Another case I investigated was plasma generation. Plasma is a partially ionized gas: this means that there are electrons not bounded to their constituent particles. It is considered to be a distinct state of matter because of its unique properties: response to electromagnetic field, electrical conductivity, ability to form beams, etc. In fact, it is the most common state of matter in the universe, due largely impart because of it composing stars and the intergalactic medium. To generate plasma, the energy and intensity being flushed into the particles must be higher than the ionization energy of the particle. As in stars and space, this energy is usually in the form of heat. Plasma is generally classified as either “hot” where a large proportion of particles are ionized or “cold” where few particles are ionized. To be classified as plasma, the charged particles must be near enough to one another to influence each other and not just neighboring particles. Terrestrial plasmas include fire, lightning, and the ionosphere.

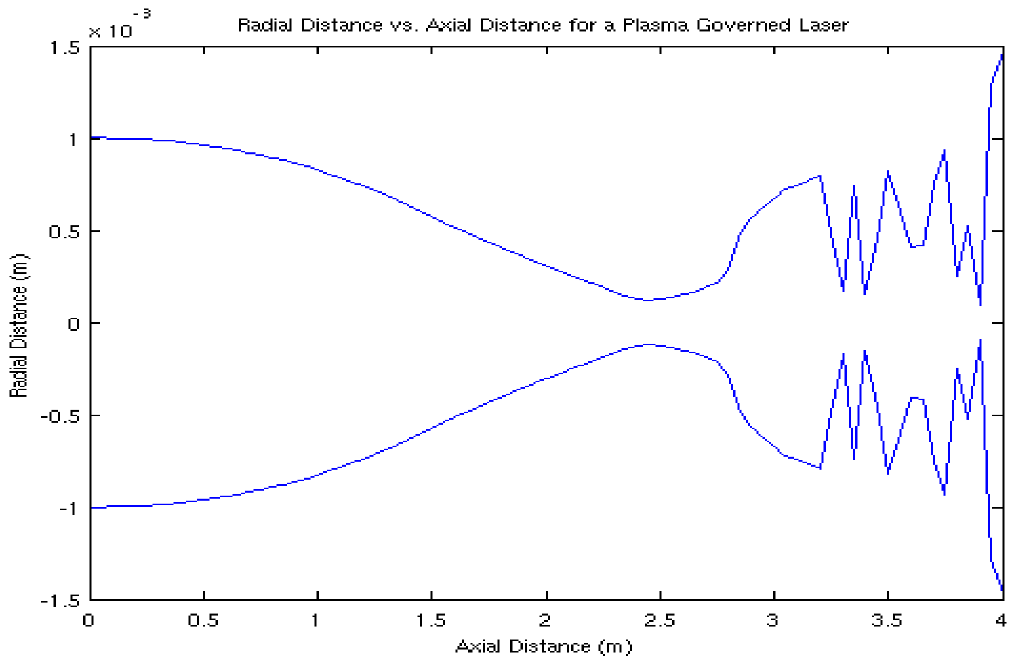
Applications of plasma include fluorescent lights, plasma torches, plasma globes, and plasma screen televisions, which work by creating voltage differences between a front and back glass plate that contains thousands of noble gas atoms: this causes the atoms to become ionized and emit photons of light.

Plasma as generated by a laser in the simulation I ran increases ionized particles exponentially over space before decaying slightly.



Here we can see how quickly ionized electrons accrue over space. In only two meters the number of ionized electrons in the system increases from a magnitude of 10^6 to 10^{22}. It should be noted that after 2.5 meters the system strays away from Gaussian behavior, thus limiting our faith in the results. In other words, we need to take the results that occur after 2.5 meters with a grain of salt.

This divergence from Gaussian behavior can be further seen in the plot of Radial Distance vs. Energy.



Again, this divergence from Gaussian conditions occurs at around 2.5 meters. We would expect the radial distance to stay fairly constant after 2.5 meters in an ideal case. By running the program and writing a simple code to run a "movie" of consecutive radial files Energy vs. Radial Distant plots shows the divergence from Gaussian conditions most clearly. Here, the lesson applied above should be restated. We cannot trust the results occurring after 2.5 meters.

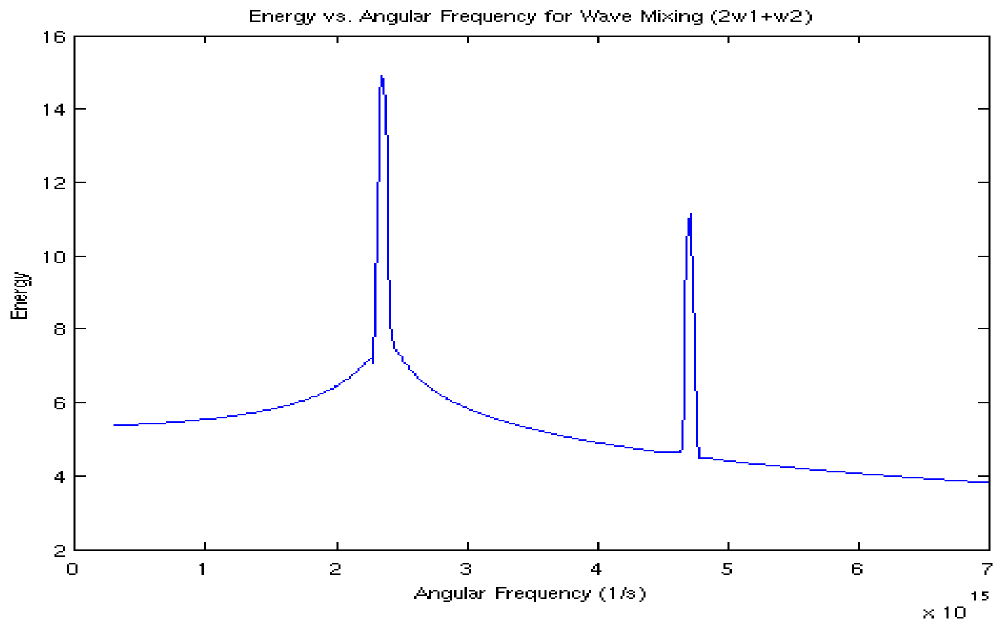
The last case I investigated was wave mixing. Here I examined the energy term of the nonlinear Schrodinger Equation and cubed it out. The original term was

$E = E_1*e^{i(w1t-k1z)} + E_2*e^{i(w2t-k2z)} + E_3*e^{-i(w1t-k1z)} +E_4*e^{-i(w2t-k2z)}$.

After cubing out the equation I examined the relationship between the exponent forms of the resulting equations. Knowing that

k = 2*pi / lambda
w = 2*pi*frequency
w = k * c (speed of light)

I was able to obtained the resulting relationship of frequencies given input wavelengths. From there I ran the program with the input wavelengths and examined the frequencies at which the most energy is outputted and compared it against my found values.



Here we can see that the highest energies occur at around $2.35*10^{15}$ and $4.92*10^{15}$. This is what the findings are through the method mentioned above.

II. Discussion and Results

I learned and applied my newfound knowledge of partial differential equations and Fourier series to laser simulation. I learned about linear-Gaussian beams, nonlinear-Kerr beams, plasma beams, wave mixing, paraxial approximation to the Helmholtz Equation, and other underlying equations to optics. I learned how to run Miro's code and find results pertaining to each of the afore-mentioned topics. Unfortunately, I did not have sufficient time to really dig deep into the project and turn around profound results.

III. Future Work

For continuation of the project, a student needs to have a knowledge base in PDE's and Fourier series. A graduate student who has had training in optics will be best suited to understand and run this code.