Project title: Fast implementation of Post's Formula for Optical Pulse Propagation in Dispersive Media

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Introduction and motivation

The purpose of this project is to consider faster, more efficient, methods for implementing the numerical Laplace transform procedure presented in paper [1]. The reference paper presents a Laplace transform based method for the evolution of optical pulses in materials with nontrivial dispersion relations. The algorithms underlying the paper are the Post inversion formula, see [3] for a calculus based description, and Faa di Bruno's formula for arbitrary order derivatives [6]. To perform the computations numerically, arbitrary precision is required. Currently, Mathematica is used. A C programming implementation using Arprec [2] may be significantly faster, particularly for the calculation of the Bell polynomials of the second kind in the Laplace transform inversion calculations.

Project goals

The aim of the project is to test the feasibility of using Arprec to perform arbitrary precision calculations of the Bell polynomials of the second kind. Toward this goal, the team members will:

a) Review the fundamentals of Maxwell's equations, Laplace transforms, Leibnitz and di Bruno's formulas, and the Bell polynomials of the second kind. In particular, extend the Matlab file exchange Bell polynomials script to include di Bruno's chain rule for higher derivatives.

b) Understand the Matlab and/or Mathematica implementations of the Bell polynomials of the second kind in Post inversion formula.

c) Write test calculations using arbitrary precision packages, such as Mathematica and Arprec, or multi-precision toolbox in Matlab.

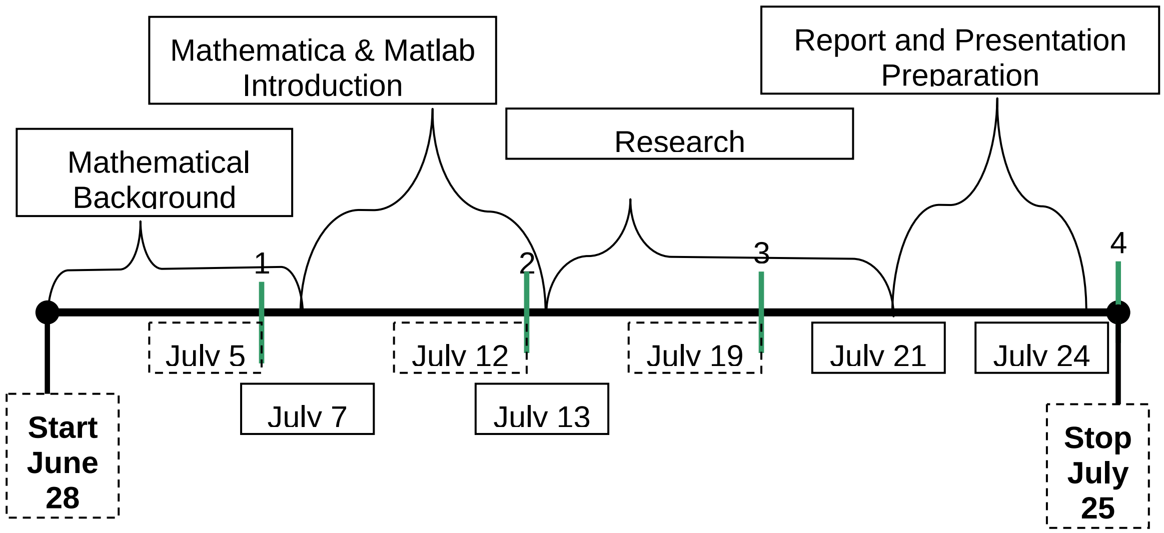
d) Translate the Bell polynomials calculation from Mathematica or Matlab to C using Aprec, or multi-precision toolbox in Matlab.

Time permitting, investigate with the Mathematica and C implementations why the “k-2” nominator in the Wynn-rho acceleration algorithm works well for sequences resulting from rational type Laplace space functions, see ref. [4]. Compare performance on various benchmark singularities [5].

Project development schedule

The project is designed to be completed in the four week period allotted. A flexible schedule, tunable to the skill set and interests of the team members, is the following:

- June 28 – July 7 : Mathematical Background
- July 7 – July 13 : Mathematica and/or Matlab and
- July 13 – July 21 : Aprec code and testing research
- July 21 – July 24 : Presentation Preparation



References

[1] P. Kano and M. Brio, "Application of Post's formula to Optical Pulse Propagation in Dispersion Media", *2009 submitted to Computers and Mathematics with Applications.*

[2] David H. Bailey, Hida Yozo, Xiaoye S. Li, and Brandon Thompson, "ARPREC: An arbitrary precision computation package" (September 1, 2002). *Lawrence Berkeley National Laboratory*. Paper LBNL-53651. http://repositories.cdlib.org/lbnl/LBNL-53651

[3] Wikipedia: "Post's inversion formula", An article at the bottom of the wiki page, "Elementary inversion of the Laplace transform", by Bryan Kurt, <http://www.rose-hulman.edu/%7Ebryan/invlap.pdf>, gives calculus based derivation of the formula.

[4] E. Weniger, "Further Discussion of Sequence Transformation Methods", http://www.nr.com/webnotes/nr3webR1.pdf

[5] J. Abate and P. P. Valko, "Multi-precision Laplace transform inversion", *Int. J. Num. Meth. Eng., 2004, vol. 60, pp. 978-93.*

[6] W. P. Johnson, The Curious History of Faa di Bruno's Formula, The American Mathematical Monthly, 2002, vol. 109, pp. 217-234.