Project title: Preconditioners for frequency domain edge FEM Maxwell solver.

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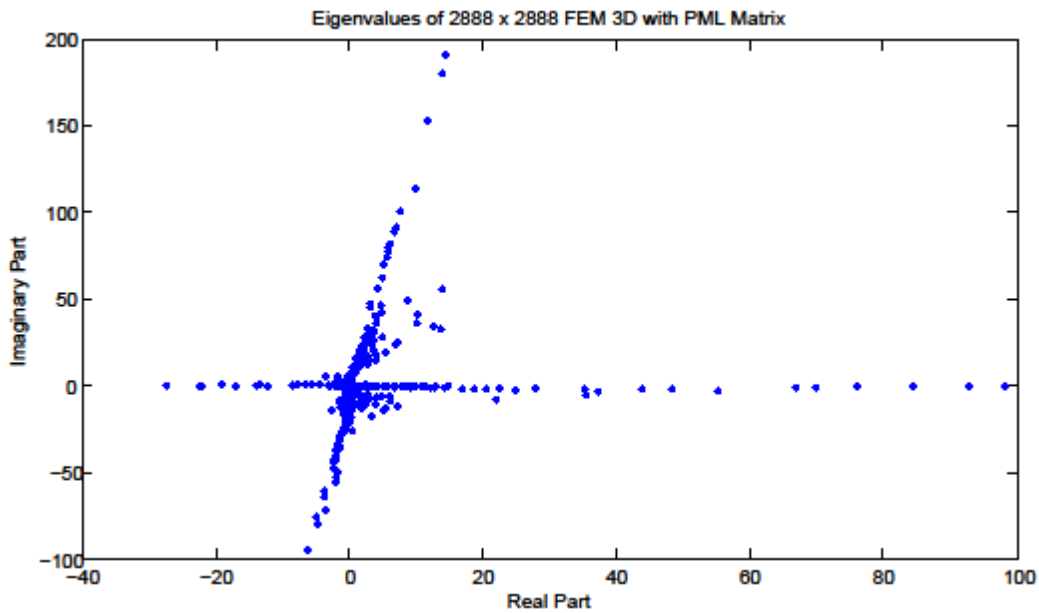
Introduction

The application of vectorial edge finite element method (VFEM) results in a sparse symmetric, but non-Hermitian, complex indefinite linear system of equations. When the number of degrees of freedom is small, the use of direct methods for the solution is feasible, however, for high order elements and/or fine meshing, a large numbers of degrees of freedom require a prohibitive amount of memory and iterative methods of solution become desirable. Preconditioning methods coupled with convergence acceleration technique aim at building efficient iterative solvers both in terms of computing time and memory requirements. A particular problem at hand characterized by extremely ill-conditioned matrices resulting to great sensitivity of the solution with respect to perturbations in the input data. In addition, the kernel of the curl operator is of infinite dimension resulting in clustering of the eigenvalues near zero which is a hallmark of slow convergence of the Krylov subspace based methods. Figure below illustrates this clustering for a small 2888x2888 FEM 3D matrix.



We are currently investigating the performance of preconditioned Krylov subspace methods. Only a select set of standard preconditioners and solvers are currently working. More sophisticated preconditioners will be researched in this proposal. See ref. [1] for detailed introduction and the current status of the project.

Project goals

The aim of the project is to compare the results of available preconditioners, analyze the properties of the system which lead to poor behavior under iterative methods, and ultimately to find, adapt, or design an effective preconditioner that allows for robust convergence and requires reasonable memory and CPU usage. Toward this goal, the team members will:

a) Explore of theoretical basis for prevalence and effectiveness of Krylov subspace-based methods in iterative solvers. Relate convergence of methods to the spectra and pseudospectra.

b) Investigate performance of various preconditioners. Consider

1. Incomplete LU preconditioning and reordering. Weigh convergence acceleration against memory demands and stability.
2. Algebra-based preconditioners in available software packages such as PETSc.
3. Adaptation of more complex edge FEM preconditioners such as in [3], [4] and [9].

c) Optimize/parallelize preconditioning and solving routines to improve effectiveness as needed.

d) Compare direct and iterative methods for solving larger systems, quantify the benefits and limitations of each.

e) Determine and quantify other factors which would affect the quality of solutions, such as system geometry, or FEM basis.

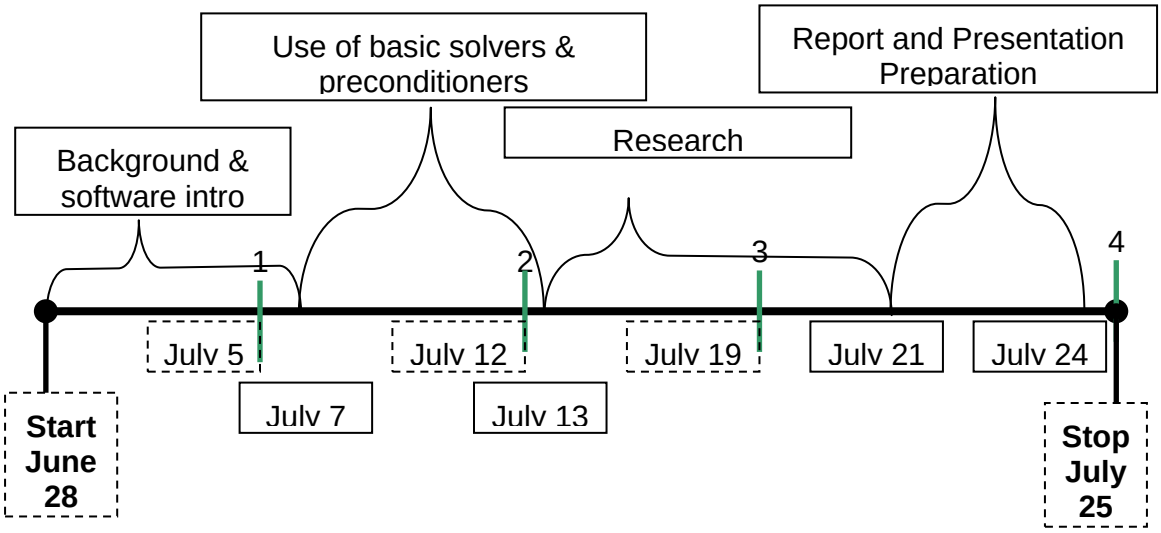
Project development schedule

In the first two weeks, the team will become familiarized with the use of linear algebra software packages and achieve a reasonable understanding of the theoretical underpinnings of preconditioning and convergence of iterative methods. During this phase the readymade preconditioning algorithms available in PETSc, MATLAB, and so forth will be tested and their effectiveness recorded.

In the second two weeks, promising preconditioning algorithms may be further developed and readied for parallelization and application on larger systems. More sophisticated FEM preconditioners such as those presented in the references section may be adapted and applied to the problem at hand. The results should be well documented and studied. The team should suggest avenues for future research.

A flexible schedule, tunable to the skill set and interests of the team members, is the following:

- June 28 – July 7: Mathematical Background, and introduction to Matlab and other software (such as PETSc).
- July 7 – July 13: Comparison of standard preconditioners and Krylov subspace methods.
- July 13 – July 21: Investigate use of more complicated preconditioners, iterative solvers, and direct solvers.
- July 21 – July 24 : Presentation Preparation



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