1. The *p.m.f.* of a finite random variable $X$ is given below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_X(x)$</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(a) Compute the mean of $X$.
(b) Compute the variance of $X$.
(c) Compute the standard deviation of $X$. Round your answer to three decimal places.

2. The *p.m.f.* of $Y$, the length (in seconds) of commercials sold by a radio station, is given below.

<table>
<thead>
<tr>
<th>$y$</th>
<th>15</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_Y(y)$</td>
<td>0.125</td>
<td>0.795</td>
<td>0.054</td>
<td>0.008</td>
<td>0.018</td>
</tr>
</tbody>
</table>

(a) Find the *c.d.f.* of $Y$.
(b) Find $\mu_Y$.
(c) Find $V(Y)$.
(d) Find $\sigma_Y$.

3. The *p.m.f.* of $T$, the length (in seconds) of a commercial sold by a local radio station, is given below.

<table>
<thead>
<tr>
<th>$t$</th>
<th>15</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_T(t)$</td>
<td>0.125</td>
<td>0.795</td>
<td>0.055</td>
<td>0.010</td>
<td>0.015</td>
</tr>
</tbody>
</table>

(a) Find the *c.d.f.* of $T$.
(b) Find the probability that the length of a commercial sold by this radio station is at least 90 seconds.
(c) Find the mean of $T$.
(d) Find the variance of $T$.
(e) Find the standard deviation of $T$. 

4. The p.m.f. of a finite random variable $Y$ is given below.

<table>
<thead>
<tr>
<th>$y$</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_Y(y)$</td>
<td>0.2</td>
<td>0.4</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(a) Find $P(Y = 5)$.
(b) Find $P(1 \leq Y < 5)$.
(c) Find $\mu_Y$.
(d) Find $\sigma_Y$.

A random sample of eight observations of $Y$ is given below.

2, 1, 5, 10, 1, 10, 5, 2

(e) Find $\bar{Y}$.
(f) Find $s$.

5. Let $X$ be a finite random variable with the p.m.f. given below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_X(x)$</td>
<td>0.4</td>
<td>0.2</td>
<td>0.1</td>
<td>?</td>
</tr>
</tbody>
</table>

(a) Find a value for $f_X(8)$ that makes this a valid probability model.
(b) Find the mean of $X$.
(c) Find the standard deviation of $X$. 

6. Let $W$ be a binomial random variable with parameters $n = 20$ and $p = 0.10$.

(a) Fill in the information that would be needed to have the Excel function $\text{BINOMDIST}$ compute $P(W \leq 4)$.

![Excel function BINOMDIST](image)

(b) Compute the mean of $W$.
(c) Compute the variance of $W$.

7. Let $Y$ be a binomial random variable with parameters $n = 20$ and $p = 0.10$.

(a) Fill in the information that would be needed to have the Excel function $\text{BINOMDIST}$ compute $P(Y = 5)$.

![Excel function BINOMDIST](image)

(b) Compute the mean of $Y$.
(c) Compute the standard deviation of $Y$. 
8. It is claimed that 76% of the residents of Tucson have at least one credit card. You plan to randomly select 5 residents of Tucson and ask them if they have at least one credit card. Let $Y$ be the number of residents who say yes.

(a) What type of random variable is $Y$?
(b) How many residents would you expect to say yes?
(c) Use the following table to find $P(Y = 3)$.

<table>
<thead>
<tr>
<th>$y$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_Y(y)$</td>
<td>0.0008</td>
<td>0.0134</td>
<td>0.09325</td>
<td>0.34610</td>
<td>0.74645</td>
<td>1.000</td>
</tr>
</tbody>
</table>

9. Approximately 62% of students who enroll in Math 115b have a 3.3 grade point average or higher; the rest do not. Let $G$ be the binomial random variable that gives the number of Math 115b students with a 3.3 grade point average or higher. Assume that 700 students are enrolled in Math 115b this year.

(a) Find $E(G)$.
(b) Find $V(G)$.
(c) Find $\sigma_G$.

10. Let $X$ be a finite random variable with the p.m.f. given below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_X(x)$</td>
<td>0.4096</td>
<td>0.4096</td>
<td>0.1536</td>
<td>0.0256</td>
<td>0.0016</td>
</tr>
</tbody>
</table>

(a) Find $\mu_X$ and $\sigma_X$.
(b) The seven observations of $X$ obtained in one experiment are 4, 3, 1, 1, 0, 1, and 4. Find the sample mean and sample standard deviation.
(c) Consider the sample mean, $\bar{x}$, as calculated in Part (b) to be a random variable. Find the mean and standard deviation of this random variable.
11. Let \( X \) be a binomial random variable with \( n = 4 \) and \( p = 0.5 \).

(a) Complete the following table for the c.d.f. of \( X \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( F_X(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0625</td>
</tr>
<tr>
<td>2</td>
<td>0.3125</td>
</tr>
<tr>
<td>3</td>
<td>0.6875</td>
</tr>
<tr>
<td>4</td>
<td>0.9375</td>
</tr>
</tbody>
</table>

(b) Create a table for the p.m.f. of \( X \).

(c) Find \( P(X = 2) \) and \( P(X = 1.5) \).

(d) Find \( P(X > 1) \) and \( P(1 \leq X \leq 3) \).

(e) Find \( E(X) \) and \( V(X) \).

(f) Find \( E(X - \mu_X) \).

(g) Find \( P(S \geq 1) \) where \( S \) is the standardization of \( X \).

A random sample of 5 observations of \( X \) is given below:

\[ 3, 2, 3, 0, 1 \]

(h) Find the mean and standard deviation of the sample.

Let \( \bar{x} \) be the random variable that represents the mean of \( X \) for samples of size 5. Let \( S_5 \) be the standardization of \( \bar{x} \) for samples of size 5.

(i) Find the mean and standard deviation of \( \bar{x} \).

(j) Find the mean and standard deviation of \( S_5 \).
12. The p.d.f. and c.d.f. of a continuous random variable, $X$, are given by

$$F_X(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\frac{x^2}{4} & \text{if } 0 \leq x \leq 2 \\
1 & \text{if } 2 < x
\end{cases}$$

and

$$f_X(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\frac{x}{2} & \text{if } 0 \leq x \leq 2, \\
0 & \text{if } 2 < x
\end{cases}$$

and plots of the two functions are shown below.

(a) Is Plot A or Plot B the graph of $F_X$? Explain.
(b) On the plot of $f_X$, shade the region whose area corresponds to $P(0.8 \leq X \leq 1.6)$.
(c) Use the region from Part (b) to estimate $P(0.8 \leq X \leq 1.6)$. Note that the area of each grid square is 0.04 square units.
(d) Use the formula for the c.d.f. of $X$ to compute $P(0.8 \leq X \leq 1.6)$ exactly.
(e) Use the graph of $f_X$ to estimate $\mu_X$.
(f) Set up and evaluate an integral that computes $P(0.8 \leq X \leq 1.6)$. Round your answer to three decimal places.
(g) Set up and evaluate an integral that computes $E(X)$. Round your answer to three decimal places.
(h) Set up and evaluate an integral that computes $V(X)$. Round your answer to three decimal places. (You will need to use the value of $E(X)$ that you computed in Part (g).)

13. The p.d.f. and c.d.f. of $T$, the weekly CPU time (in hours) used by an accounting firm, are given below.

$$f_T(t) = \begin{cases} 3 & \text{if } 0 \leq t \leq 1 \\ \frac{1}{64}t^2(4-t) & \text{if } 0 < t < 4 \\ 0 & \text{if } t > 4 \end{cases}$$

$$F_T(t) = \begin{cases} 0 & \text{if } t < 0 \\ \frac{1}{256}t^3(16 - 3t) & \text{if } 0 \leq t \leq 4 \\ 1 & \text{if } t > 4 \end{cases}$$

(a) Set up, but do not evaluate, an integral that gives the probability that the CPU time used by the firm in a given week is at least 90 minutes.
(b) Use the c.d.f. to find the probability that the CPU time used by the firm in a given week is at least 90 minutes.

14. The p.d.f. of $T$, the proportion of erroneous tax returns filed with the IRS in a year, is given below.

$$f_T(t) = \begin{cases} 90t(1-t)^8 & \text{if } 0 \leq t \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Set up, but do not evaluate, an integral that could be used to verify that this is a valid p.d.f.
(b) Set up, but do not evaluate, an integral that gives the probability that the proportion of erroneous tax returns filed with the IRS in a year is between 0.10 and 0.20.
(c) Set up, but do not evaluate, an integral that gives the expected value of $T$.
(d) Fill in the information that would be needed in order to use Integrating.xls to verify that the expected value of $T$ is 0.1818.

<table>
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<tr>
<td>Formula for $f(x)$</td>
<td>$x$</td>
<td>$f(x)$</td>
<td>$A$</td>
</tr>
</tbody>
</table>

(e) Set up, but do not evaluate, an integral that gives the variance of $T$. 
15. The p.d.f. of a continuous random variable, $X$, is given below.

\[ f_X(x) = \begin{cases} 
0 & \text{if } x < -1 \\
\frac{x^2}{3} & \text{if } -1 \leq x \leq 2 \\
0 & \text{if } x > 2 
\end{cases} \]

(a) Set up, but do not evaluate, an integral that gives $E(X)$.
(b) Set up, but do not evaluate, an integral that gives $V(X)$. Assume that $E(X) = 1.25$.
(c) Set up, but do not evaluate, an integral that gives $P(X \leq 1)$.

16. The plots of six p.d.f.’s are given below.

\begin{enumerate}[(a)]
\item Which one(s) could correspond to a standard normal random variable?
\item Which one(s) could correspond to a uniform random variable?
\item Which one(s) could correspond to an exponential random variable?
\item Which one(s) could not possibly correspond to a normal random variable?
\item Which one could correspond to a normal random variable with $\mu_X = 5$ and $\sigma_X = 3$?
\end{enumerate}
17. The graphs of four p.d.f.'s are given below.

(a) Which one could be the graph of the p.d.f. of an exponential random variable?
(b) Which one could be the graph of the p.d.f. of the standard normal random variable?
18. The graphs of four functions are shown below:

(i)

(ii)

(iii)

(iv)

(a) Which of the graphs could be p.d.f.’s?
(b) Which of the graphs could be c.d.f.’s?
(c) Match each p.d.f. with its possible c.d.f.

19. Evaluate the integrals given below.

(a) \[ \int_{0}^{\infty} \frac{1}{20} e^{-x/20} \, dx \]

(b) \[ \int_{0}^{\infty} x \cdot 0.05 e^{-0.05x} \, dx \]

(c) \[ \int_{0}^{\infty} (x - 20)^2 \cdot \frac{1}{20} e^{-x/20} \, dx \]

(d) \[ \int_{0}^{25} \frac{1}{20} e^{-x/20} \, dx \]
20. Let \( f(x) = \begin{cases} 
0 & \text{if } x < 1 \\
0.2 & \text{if } 1 \leq x \leq 6 \\
0 & \text{if } x > 6 
\end{cases} \).

(a) Find \( \int_{2}^{5} f(x) \, dx \).

(b) Find \( \int_{4}^{9} f(x) \, dx \).

(c) If \( f(x) \) is a p.d.f., what does your answer in Part (a) represent?

21. Match the expressions on the left with their values on the right. A value may be used more than once.

(i) \( \int_{0}^{\infty} \frac{1}{5} e^{-x/5} \, dx \) \hspace{1cm} A. 5

(ii) \( \int_{0}^{\infty} x \cdot \frac{1}{5} e^{-x/5} \, dx \) \hspace{1cm} B. 25

(iii) \( \int_{0}^{\infty} (x-5)^2 \cdot \frac{1}{5} e^{-x/5} \, dx \) \hspace{1cm} C. 1

(iv) \( \int_{-1}^{4} 0.2 \, dx \) \hspace{1cm} D. 1.5

E. 0.2

22. Let \( X \) be an exponential random variable with \( E(X) = 3 \), and let \( S \) be the standardization of \( X \). Compute \( P(0 \leq S \leq 4) \).

23. Suppose that \( R \) is a random variable with a mean of 31.5 and a variance of 203.9, and let \( S \) be the standardization of \( R \). Find the value \( b \) such that \( P(S < 1) = P(R < b) \).

24. Let \( X \) be a random variable that is uniform on the interval \([0,5]\).

(a) Find a formula for \( f_X(x) \).

(b) Find a formula for \( \int_{0}^{x} f_X(u) \, du \).

(c) Find the derivative of \( \int_{0}^{x} f_X(u) \, du \) with respect to \( x \).
25. Find \( \int_0^x 0.2e^{-0.2u} \, du \). (Your answer will be a function.)

26. Find \( F'(x) \), if \( F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-x/5} & \text{if } x \geq 0 \end{cases} \).

27. Six observations of a random variable \( X \) are given below.

\[
10.3, 12.4, 8.9, 10.3, 9.0, 11.8
\]

(a) Compute the sample mean.
(b) Compute the sample variance.
(c) Compute the sample standard deviation. Round your answer to three decimal places.

28. The price-to-earnings ratios for six stocks selected at random from all of the stocks traded on the New York Stock exchange are given below.

\[
7, 20, 18, 17, 20, 32
\]

(a) Find the sample mean.
(b) Find the sample variance.
(c) Find the sample standard deviation.

29. Let \( T \) be the random variable giving the number of hours per week that a randomly selected student spends on a computer. Eight observations of \( T \) are given below.

\[
14, 22, 5, 10, 20, 9, 10, 24
\]

(a) Use this information to approximate the probability that a randomly selected student spends more than 20 hours per week on a computer.
(b) Use this information to approximate the expected value of \( T \).
(c) Use this information to approximate the standard deviation of \( T \).

30. Let \( X \) be the random variable which gives the number of customers who visit your business in a given day. You know that the parameters of \( X \) are \( \mu_X = 30 \) and \( \sigma_X = 6 \), but you do not know the p.d.f. or the c.d.f. for \( X \). Let \( \bar{x} \) be the random variable that is the mean of a random sample of size \( n = 80 \) days.

(a) Compute \( \mu_{\bar{x}} \).
(b) Compute \( V(\bar{x}) \).
(c) Compute \( \sigma_{\bar{x}} \).
(d) Give a formula for the random variable, \( S \), that is the standardization of \( \bar{x} \).
(e) What is the approximate distribution of \( S \)?
31. A normal random variable $X$ gives the number of ounces of soda in a randomly selected can from a given canning plant. It is known that the mean of $X$ is close to 12 ounces and that $\sigma_X = 0.4$ ounces. A plot of $f_X$ is show below.

Let $\bar{x}$ be the mean of a random sample of size $n = 4$ soda cans.

(a) Compute $\sigma_{\bar{x}}$.
(b) Sketch a graph of the probability density function for $\bar{x}$ on the above plot.
(c) Use standard deviations to explain why the mean of a sample of size $n = 16$ cans would be likely to give a better estimate for $\mu_X$ than would the mean of a sample of size $n = 4$ cans.

32. Let $X$ be a random variable, which can assume only the values of 0 and 1, with $P(X = 0) = 0.7$ and $P(X = 1) = 0.3$. Let $S_2$ be the standardization of $\bar{x}$ for sample sizes of $n = 2$.

(a) Compute the mean and standard deviation of $\bar{x}$.
(b) Compute all values for the $p.m.f.$ of $S_2$.
(c) Compute the mean and standard deviation of $S_2$.

33. Let $X$ be the random variable which gives the time that it takes for an employee of a company to learn a new task. It has been determined that the mean of $X$ is 6 hours and that the standard deviation of $X$ is 1.6 hours. Let $\bar{x}$ be the random variable that is the mean of a random sample of size $n = 50$ employees.

(a) Find $\mu_{\bar{x}}$.
(b) Find $V(\bar{x})$.
(c) Find $\sigma_{\bar{x}}$.
(d) Give a formula for the random variable, $S$, that is the standardization of $\bar{x}$.
(e) What is the approximate distribution of $S$?
34. Scores on the Graduate Management Admissions Test (GMAT) are normally distributed with mean 528 and standard deviation 112. Let $X$ be the score obtained on the GMAT by a randomly selected student.

(a) Find a formula for the $p.d.f.$ of $X$.
(b) Draw the graph of the $p.d.f.$ of $X$ and shade the area that corresponds to the probability that the score on the GMAT is less than 500.
(c) Fill in the information that would be needed in order to use Integrating.xls to estimate the probability that the score on the GMAT is less than 500.

<table>
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<td>$x$</td>
<td>$f(x)$</td>
<td>$A$</td>
</tr>
</tbody>
</table>

(d) Fill in the information that would be needed in order to have the Excel function $\text{NORMDIST}$ compute the probability that the score on the GMAT is less than 500.

$\text{NORMDIST}$

$\text{X}$ = $\text{number}$

$\text{Mean}$ = $\text{number}$

$\text{Standard_dev}$ = $\text{number}$

$\text{Cumulative}$ = $\text{logical}$

Returns the normal cumulative distribution for the specified mean and standard deviation.

$X$ is the value for which you want the distribution.

Formulas result =

OK  Cancel
35. Let \( X \) be the score obtained on the Graduate Management Admissions Test (GMAT) by a randomly selected student. Then \( X \) has a normal distribution with mean 528 and standard deviation 112.

(a) Find the formula for the \( p.d.f. \) of \( X \).
(b) Set up, but do not evaluate, an integral that gives the probability that the score on the GMAT is less than 680.
(c) Fill in the information that would be needed to have the \textit{Excel} function \texttt{NORMDIST} compute the probability that the score on the GMAT is less than 680.

\[
\text{NORMDIST} \begin{array}{c}
\text{X} \\
\text{Mean} \\
\text{Standard\_dev} \\
\text{Cumulative} \\
\end{array} = \begin{array}{c}
\text{number} \\
\text{number} \\
\text{number} \\
\text{logical} \\
\end{array}
\]

Returns the normal cumulative distribution for the specified mean and standard deviation.

\( X \) is the value for which you want the distribution.

(d) Find the mean of the distribution of the average score obtained by a sample of 100 randomly selected students.
(e) Find the standard deviation of the distribution of the average score obtained by a sample of 100 randomly selected students.

36. Commute times in New York State are normally distributed with a mean of 28 minutes and a standard deviation of 9 minutes. Let \( X \) be the time, in minutes, that it takes a randomly selected New Yorker to get to work on a randomly selected day.

(a) Write the formula for the \( p.d.f. \) of \( X \).
(b) Sketch the graph of the \( p.d.f. \).
(c) Set up, but do not evaluate, an integral that gives the probability that the travel time for a randomly selected New Yorker on a randomly selected day is at least 37 minutes.
(d) Represent the integral in Part (c) on the graph of the \( p.d.f. \).
(e) Give the exact \textit{Excel} formula that would be used to calculate \( P(X \leq 30) \).

37. Let \( M \) be the normal random variable that gives the starting salary for a graduate from the school of business. Assume that \( \mu_M = 38,142 \) and \( \sigma_M = 6,595 \).

(a) Give the expression for the \( p.d.f. \) of \( M \).
(b) If \( P(-1.44 < Z < 1.44) = 0.85 \), find numbers \( a \) and \( b \) such that \( P(a \leq M \leq b) = 0.85 \).
38. Suppose that $X$ is a continuous random variable with unknown mean and standard deviation $\sigma_X = 150$. The mean of a sample of 100 observations of $X$ is 425.

(a) Find the standard deviation of $\bar{x}$.
(b) Use the fact that $P(-1.96 \leq Z \leq 1.96) = 0.95$ to find a 95% confidence interval for $\mu_X$.

39. The scores on a Math 115b test were approximately normally distributed. Your friend scores 68 points on the test. He wants to assure his advisor that this is a decent score based on the scores of the other students, so he asks 5 friends in class what they scored on the test. He calculates that the mean of these 5 scores is 72.4 and that the standard deviation is 8.2. If $P(-1.96 < Z < 1.96) = 0.95$, find and interpret a 95% confidence interval for the mean of all scores.

40. Let $V$ be the random variable that gives the value of an oil lease similar to the one which is to be auctioned. A random sample of 36 historical values are used to estimate the mean of $V$. The sample mean is $93$ million and the sample standard deviation is $13.4$ million. If $P(-1.645 < Z < 1.645) = 0.90$, find and interpret a 90% confidence interval for $\mu_V$.

41. Let the random variable $X$ represent the time (in hours and parts of hours) needed for a business student to complete the final exam. For 90 students, the sample mean was $\bar{x} = 1.6$, and the sample standard deviation was $s = 0.2$. If $P(-1.96 < Z < 1.96) = 0.95$, find and interpret a 95% confidence interval for $\mu_X$.

42. The graph of the p.d.f. for the standard normal random variable is given below. If the shaded area is 0.1359, find $P(-2 \leq X \leq -1)$.

43. A company that produces snack food has calibrated its packaging machines to produce bags of potato chips with an average weight of 588 grams with a standard deviation of 12 grams. Assume that the weights of the bags are normally distributed.

(a) Explain what is meant by a standard deviation of 12 grams in terms of this example.
(b) What is the probability that the weight of a randomly selected bag exceeds 600 grams?
44. Let \( X \) be a normal random variable with a mean of 4 and a standard deviation of 2. The graph of the \( p.d.f \) of \( X \) is given below. Use this graph and your knowledge of the standard normal random variable to answer the following questions.

Find the following.

(a) \( P(4 \leq X \leq 6) \)
(b) \( F_X(6) - F_X(2) \)
(c) \( \int_2^6 f_X(x) \, dx \)
(d) \( F_X(4) \)
(e) \( f_X(4) \)
(f) \( \text{NORMDIST}(4,4,2,\text{FALSE}) \)
(g) \( f_X(4) - f_X(2) \)

45. Which of the following Excel formulas would return a randomly generated observation of a normal random variable with a mean of 25 and a standard deviation of 3?

(A) \( =\text{NORMDIST}(\text{RAND}(),25,3,\text{TRUE}) \)
(B) \( =\text{NORMDIST}(\text{RAND}(),3,25,\text{TRUE}) \)
(C) \( =\text{NORMDIST}(\text{RAND}(),25,3,\text{FALSE}) \)
(D) \( =\text{NORMINV}(\text{RAND}(),25,3,\text{TRUE}) \)
(E) \( =\text{NORMINV}(\text{RAND}(),25,3) \)
46. Which of the following is/are true of a general normal random variable?

(i) It has a p.d.f. that is symmetrical around the mean.
(ii) It has a c.d.f. that is symmetrical around one.
(iii) It is a generalization of the standard normal random variable.

(A) (i) only  
(B) (ii) only  
(C) (i) and (ii) only  
(D) (i) and (iii) only  
(E) (ii) and (iii) only

47. Which of the following is/are true of the p.d.f. of a general normal random variable?

(i) It can be used to calculate probabilities.
(ii) It is the derivative of the c.d.f.
(iii) It can be obtained using NORMDIST.

(A) (i) only  
(B) (ii) only  
(C) (iii) only  
(D) (i) and (ii) only  
(E) (i), (ii), and (iii)
48. Let $X$ be a normal random variable with $\mu_X = 24$ and $\sigma_X = 3.2$. Fill in the information that would be needed to have the Excel function Random Number Generation create random values of $X$ in Cells A1:F10.

![Random Number Generation dialog box](image)
49. Let $X$ be a normal random variable with mean $\mu_X = 20$ and standard deviation $\sigma_X = 5$, and let $Y$ be a normal random variable with mean $\mu_Y = 75$ and standard deviation $\sigma_Y = 10$. Fill in the information that would be needed to have Excel create fixed random values of $X$ in Cells A1:A10 and a changeable random value of $Y$ in Cell B1.

![Random Number Generation dialog box]

![NORMINV function]

Returns the inverse of the normal cumulative distribution for the specified mean and standard deviation.

**Probability** is a probability corresponding to the normal distribution, a number between 0 and 1 inclusive.

Formula result =
50. Fifteen companies all bid on oil leases. The following data is a small part of the records on past bids. All monetary amounts are in millions of dollars.

<table>
<thead>
<tr>
<th>Leases</th>
<th>Signals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proven Value</td>
<td>Company 1</td>
</tr>
<tr>
<td>$103.3</td>
<td>$99.0</td>
</tr>
<tr>
<td>$109.5</td>
<td>$91.7</td>
</tr>
<tr>
<td>$98.7</td>
<td>$105.8</td>
</tr>
</tbody>
</table>

(a) Compute the mean error in the signals.
(b) Let $R$ be the continuous random variable that gives the error in a geologist's estimate for the value of a lease. Experience allows us to assume that $R$ is normal with $\mu_R = 0$ and $\sigma_R = 10$ million dollars. Suppose that the 15 companies form 3 bidding rings of equal sizes. Let $M$ be the random variable giving the mean of the errors for a set of signals for the companies in one of the bidding rings. Compute the standard deviation of $M$. Round your answer to three decimal places.

51. This problem refers to Project 2. The following table shows the signals for several companies from two previous auctions. All monetary amounts are in millions of dollars.

<table>
<thead>
<tr>
<th>Leases</th>
<th>Signals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proven Value</td>
<td>Company 1</td>
</tr>
<tr>
<td>$55</td>
<td>$52</td>
</tr>
<tr>
<td>$120</td>
<td>$113</td>
</tr>
</tbody>
</table>

(a) What is a signal?
(b) Complete the following table:

<table>
<thead>
<tr>
<th>Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company 1</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

(c) Find the mean and standard deviation of the errors in Part (b).
(d) In theory, what should the mean of the errors be? Why?

52. Explain the five random variables used in the auction project: $V$, $S_V$, $R$, $C$, and $B$.

53. What assumption in the auction project allows you to assume that $E(R) = 0$?
54. The table given below shows the payoffs (in dollars) for a game in which two players are competing against each other. Player 1’s payoff is given in the lower left-hand corner of each square, and Player 2’s payoff is given in the upper right-hand corner of each square.

\[
\begin{array}{ccc}
 & 1 & 2 & 3 \\
A & 2 & 9 & 7 \\
 & 5 & 4 & 6 \\
B & 5 & 1 & 8 \\
 & 8 & 3 & 2 \\
C & 4 & 6 & 3 \\
 & 7 & 1 & 9 \\
\end{array}
\]

What is the Nash equilibrium of this game?

55. The table given below shows the payoffs (in dollars) for a game in which two players are competing against each other. Player 1’s payoff is given in the lower left-hand corner of each square, and Player 2’s payoff is given in the upper right-hand corner of each square.

\[
\begin{array}{ccc}
 & 1 & 2 & 3 \\
A & 1 & 3 & 6 \\
 & 7 & 8 & 5 \\
B & 2 & 4 & 5 \\
 & 3 & 1 & 6 \\
C & 9 & 8 & 7 \\
 & 4 & 9 & 2 \\
\end{array}
\]

What is the Nash equilibrium of this game?